UNIVERSITY OF EDUCATION KARLSRUHE

INSTITUTE OF MATHEMATICS AND COMPUTER SCIENCE

Improving Attitudes towards Geometric Proof through A Suggested Story-Based Dynamic Geometry Approach

A Dissertation for the Doctor of Philosophy

By

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2010

DECLARATION

I declare that the thesis hereby submitted for the Ph.D. degree at the University of Education Karlsruhe is my own work, where I have consulted the published work of others; this is always clearly indicated by special reference.

No part of this thesis has been previously submitted at another University, in Germany or overseas, for any degree or examination.

Hussein Abdelfatah

Karlsruhe, November, 2010

Acknowledgement

First and foremost I would like to thank my advisor Prof. Dr. Ulrich Kortenkamp for his hospitality, and open heart and mind. I found him not only the perfect professor to supervise my Ph.D. but also a real friend whom I will never forget after returning to my homeland. I also wish to express my thanks to all members of the University of Education Karlsruhe, where I was greeted with warm smiles every morning. I am especially grateful for Prof. Dr. Mutfried Hartmann; his judgment of the appropriateness of the measurement tools used in the present study and his support during the study experiments. I am grateful for Prof. Dr. Christian Stellfeldt for his support during the study experiments in the University of Education Karlsruhe. My thanks go further to my wonderful colleague Elena Kohler for working with me for such a long period and for her opinions during the closing stages of the study. I will also never forget the enjoyable time I spent in the office with my dear friend and colleague Christian Dohrmann.

I would also like to acknowledge all of the members of the Mathematics Department at Bayreuth University, especially Prof. Dr. Peter Baptist, where I worked for a long time developing the treatments of the present study, and which also facilitated a pilot experiment with students from the “Alexander-von-Humboldt” school. In this regard I wish to show my appreciation for the efforts of Mr. Edgar Höniger and his support during the pilot experiment at “Alexander-von-Humboldt” school. I also want to thank Prof. Dr. Alfred Wassermann and Dr. Carsten Miller for the discussions we had on translating the user interface of GEONExT into Arabic, with the result that it is now possible for students in more than twenty-two countries to use the software in their native language. I cannot also forget the brief time I spent with my colleagues Andreas Fest and Mohamed El-Demerdash.

In my home country, I gratefully acknowledge the funding sources that made the idea of my Ph.D. possible. I was funded by the Egyptian Ministry of Higher Education for 4 years, with the advantage of having my position as a lecturer in the Curricula and Instruction Department held open.

And from my home faculty, the Faculty of Education at Suez Canal University, I gratefully acknowledge Prof. Dr. Shaban Hefeny, Prof. Dr. Sami Hashem and Dr. Afaf Atia for their opinions about the learning interface of the suggested activities and design of the questionnaires.

I also appreciate the participation of all my colleagues from the Faculty of Education, Suez Canal University in my two seminar sessions in Feb. 2009 entitled “Developing a new approach based on dynamic geometry software and real-life geometry situations to improve students’ attitudes”

And last but my no means least, I would like to thank my family, my parents, brothers and my sister, my wife and my two daughters, for all their love and belief in the importance of my work.

Now, I want to thank my god “Allah” with some words from The Holy Qur’an:

“Great is the grace of Allah unto thee” (Sura/chapter 4: Al-Nisa, from Aya/Verse No. 113).

Hussein Abdelfatah
Karlsruhe, October 2010

Preface

It was the beginning of a journey, a journey to the land of science and ideas – Germany. It was a journey of difficulty and suffering through being apart from my family, but it was also a journey of new experiences and even enjoyment with new friends that I will never forget. It started in Egypt in 2006 in Ismailia when I gave a very successful in the seminar of the Curricula and Instruction Dept. where I presented a proposal for my Ph.D. research entitled "The effectiveness of a mathematics virtual laboratory on students' attitudes" and its aim was to develop a learning environment simulating mathematical tools and enabling students to conduct experimentations and explore concepts and theorems by themselves. The feedback from my colleagues and the dean of the department was positive, and after three months of work using Macromedia Director, I found by searching the internet that the main idea of the research "virtual mathematical tools" under another concept "Dynamic Geometry Software", which was still completely new, not only in Egypt but also in many other countries. At the same time, Suez Canal University and the Egyptian Ministry of Higher Education nominated me, like many other colleagues in several fields from different Egyptian universities, to choose a country such as France, Germany, United Kingdom or United States of America to embark on a Ph.D. Like many of Egyptians, I knew about Germany as being the land of ideas and the home country of Einstein. This is why I opted to go to Germany. After my arrival on Thursday, March 01, 2007 I started to make myself familiar with the new facilities at my disposal including the various types of dynamic geometry software such as Geogebra, GEONExT, Cinderella, and Euclid … etc. along with launching the present study.

Here I would like to describe my first impressions of dynamic geometry software. Rather like the way the Hubble Space Telescope has made a significant contribution to astronomy in enabling astronomers to discover hidden structures and properties of our distant universe, dynamic geometry software has allowed new worlds to become viewable and tangible in mathematics and particularly in geometry. Furthermore, I found that this software is not only able to facilitate many difficult geometric concepts and statements; it also has the power to model real-life geometric situations and apply geometric statements in daily life. In other words, it has given us the power to utilize geometric concepts and theorems in the context of our daily lives and mathematizing real-life geometry problems in learning contexts inside class.

Despite the argument that beauty is the first test, and there is no permanent place in the world for ugly mathematics (Godfrey Harold Hardy, 1877-1947), our students and even mathematics teachers are still very far from seeing this beauty. And as the Hubble Space Telescope provided us with a more powerful eye for viewing the beauty in our universe, dynamic geometry software might enable both students and teachers to see the beauty in mathematics, and in particular to realize many relevant geometrical phenomena “e.g., proprieties” to real life. Unfortunately, this technology is still scarcely used in Egypt and even unknown in some other countries around the world where only a limited number of related studies exist.

In this modest work, I tried to investigate one of the most unwanted emotions regarding mathematics, “I hate mathematics” – the common belief that mathematics is not important outside mathematics classes and the common students’ expression “I am not a math person”.

Also, this work suggests a new organization for geometric content that could be so-called “Story-Based Geometry” and based on one of the most effective channels, “dynamic geometry software”, for transferring our message “Mathematics as Knowledge and Value” to our customers “The Students”

Last, but not least, I hope that my brief and modest words will have some benefits for all of my dear colleagues and future students, as words can live longer than humans.

Hussein Abdelfatah
Karlsruhe, October 2010

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1 Introduction

For years, many attempts either by researchers in prior studies or by specialists, in the field of mathematics education, have sought to improve various educational variables such as students' achievements, performances or competencies using several treatments. Nevertheless, how can we hope to instill into students a greater mathematical knowledge and improve their competencies, e.g., their ability to do mathematics, while they do not have a positive attitude towards it or, in other words, are not willing to do mathematics. This might add more cognitive load which may lead to a vicious circle of negative attitudes and learning difficulties.

According to the review of related literature, in the traditional geometry class, particularly in Egypt, the teacher's role is to introduce geometric concepts and theorems on the board and in the front of the class without any active contributions from students in formulating new knowledge. This does not show appreciation for their minds and abilities. Further, some students are aware that the geometric theorems can be found in textbooks, and that they are the work of famous geometers! Thus, they are obviously true, which might lead a student to state "*I am convinced by their trueness*". Consequently, the false conception in geometry class regarding the function of geometric proof as just being to convince the reader of the correctness of theorems could be the reason behind a question that arises in students' minds: "*Why do we need to prove?*" In addition, the present researcher observed through interviews and discussions the positive attitudes of students towards courses such as languages, biology, geology... etc., that most students also describe these courses as being interesting, and that they have a tangible use in the real world. In contrast, they are asking themselves: "*Why do we need to learn geometry? When will we need to use Pythagoras or Thales theorems?*" This shows that they have not gone through a learning experience that has enabled them to realize the importance of geometry in everyday life. Therefore, what should now be considered are the learners' beliefs about geometry and geometric proof, especially when the teacher mandates them to be used to verify the trueness of a theorem.

Furthermore, while working at the Faculty of Education in Suez Canal University in Egypt, the present researcher faced some students' criticisms that the mathematics courses do not fit their needs as future mathematics teachers, as they have to study many mathematical branches that they will never teach in school after graduating.

Moreover, they criticized that they are given no plane geometry courses and even no instruction in how to teach it, when later they will have to teach it in school. The university officials responded to these criticisms by saying that students are accepted to the Faculty of Education according to their marks in their final exam in high school together with entrance interviews, and it is assumed that these courses have already been studied so students will now be ready to go deeper into studying other mathematics branches.

In contrast, the researcher found during his Ph.D. scholarship in Germany that plane geometry and its didactics are essential branches for students who are preparing to be mathematics teachers. However, according to an entrance test for more than 200 German students as well as indirect interviews with students, a wide range of German students did not have the basic geometrical knowledge that should be achieved before arriving at the mathematics section in Faculties of Education, in addition to their false beliefs about the importance of geometry and the functions of geometric proof.

Although, the National Council of Teachers of Mathematics 'NCTM' in the U.S.A. stated that geometry is a natural place for the development of students' reasoning and justification skills, where students should understand that a part of the beauty of mathematics is that when interesting things happen it is usually for a good reason (NCTM, 2000, p. 42), unfortunately, a wide range of students and even teachers around the world are unable to appreciate the beauty in mathematics, as the findings of many studies show that students in geometry classes have neither the experience to see the importance of geometry in daily life nor to see the reasons for doing geometric proof (Chazan, 1993; Almeida, 2000; Refaat, 2001; Nordström, 2003; Gfeller, 2005), Kemeny (2003) stated that students see proofs as an unnecessary procedure to arrive at a simple truth that they already know and accept.

In the United States, proof has been an important goal of the geometry curricula for more than a century (Senk, 1989, p. 309), and also in Egypt it plays a key role in helping students to construct mathematical structures (Refaat, 2001, p. 63). And despite reasoning being defined as several interrelated processes categorized as sense-making, conjecturing, convincing, reflecting, generalizing and justifying (Bjuland, 2002), the current situation in Egypt shows that asking students to write a formal proof is viewed as the main, even the first task of geometry class, which aims at verifying the correctness of a theorem followed by posing further exercises and solving them.

In contrast, most German curricula accepted that rigorous proofs should be avoided. Rather, it recommends that students should try out ideas, observe, guess and make reasonable justifications (Kwak, 2005, p. 14, p. 37). And according to one well-known theory of geometrical thinking levels, van Hiele defined five levels of geometrical thinking starting with visual recognition, going through analysis, informal deduction, with the two higher levels being deduction and rigor. Van Hiele concluded that learners cannot pass over one level to reach a higher one (De Villiers, 2006). Olkun et al. (2005) agreed to some extent with van Hiele, stating that a good geometry lesson is considered to be accessible to all students, allowing them to start from their own level of development.

In the present study, it was supposed that skipping one or more of the geometrical thinking levels or one of the reasoning processes could not only reduce the opportunities to improve the reasoning skills, but could also affect students' attitudes toward geometry and especially towards geometric proof. In other words, this study aimed to investigate how to improve another domain, which alongside achievement and geometric thinking levels, considers that the direct presentation of formal proof in traditional geometry class could add additional false beliefs and negative emotions towards geometry and geometric proof.

Hanna (1993) and Hersh (1993) described that the proof has two roles: one to convince that deals with what is true and another to explain that deals with why it is true. In view of that, Hersh (1993, p. 396) differentiated between the role of proof in geometry class, and its role in research. While "… *In the classroom, convincing is no problem, students* *are easily convinced*" he argued that the effect of proof-learning experiences on students is often more emotional than intellectual and stated that "*if the instructor gives no better reason for proof than, that's math! How will the students find out why or wherefore, except: that's math!*"

According to the above indications, many related studies were conducted with the aim of changing students' negative attitudes. Actually, most of these studies accepted a "one-dimensional" definition of attitudes related only to students' emotions and feelings toward learning subjects and situations. Thus, the present study is based on a more systematic definition of attitudes as multiple components including students' beliefs as an input component for this system and students' behaviors such as their participations and roles, where the third component is students' emotions that could be negative or positive according to their beliefs, their participations and roles in geometry class.

While there was agreement around the world on the importance of geometry and geometric proof regardless of the diversity of educational systems, another question remains:

> "How could the new trends of mathematics instruction play a part in changing students' attitudes?"

The National Council of Teachers of Mathematics in the U.S.A. presented possible answer to the question and reported that everyone can use geometric ideas to represent and solve problems in the real world and also that studying measurement is important because of its practicality in so many aspects of everyday life. Furthermore, the report lays emphasis on the importance of recognizing, applying and making connections between mathematics and real-world contexts (NCTM, 2009). This answer is also consistent with the principles of the constructivism theory, as Sun and Williams (2003) referred to the importance of constructivist learning in acquiring knowledge that required learner-centered, goal-directed, and real-life problems in order to find meaningful solutions. Kanuka and Anderson (1999) considered the importance of the learners' prior knowledge in applying their understanding in learning activities that have real-world relevance for them. This obviously showed the role of real-life-centered learning in order to stimulate students to actively participate in geometry class.

Therefore, the present study sought to engage students in real-life geometry situations, which could change their beliefs about geometry from just consisting of learning tasks inside the classroom to feeling its importance in real life, and to feeling that doing geometry and geometric proof are not just professional work limited to mathematicians.

However, to complete the answer, designing instructional environments that evolve students' experiences from just recognizing real-life geometry situations to the level of doing geometry and proving becomes the challenge. Many studies have been conducted to investigate the facilities and the effectiveness of using several learning environments to overcome the inadequacy in mathematics teaching and learning, especially in geometry (e.g., Noss, 1988; Refaat, 2001). However, studies that engage students in real-life geometry situations to overcome difficulties in geometry and geometric-proof learning as well as improving the attitudes towards these subjects in mathematics classrooms are scarce (e.g., Stillman, 2006; Pierce & Stacey, 2006; Duatepe-Paksu; Ubuz, 2009). Obviously, developing a geometry-learning environment that enables students to participate in hands-on activities is needed in the context of the present study. For that reason, dynamic geometry software was chosen as a means to present geometric concepts and theorems in real-life situations, since it may provide the students with an appropriate learning environment, in which they can feel the possibility of doing geometry, formulating geometric statements by themselves and realizing the relevancy of geometry in real life. The standards set by the National Council of Teachers of Mathematics support the idea of the combination of dynamic geometry software and real-life geometry and emphasize the use of modeling to explore geometric ideas and realize their usability in real-world contexts (NCTM, 2009).

1.1 Purpose of the Study

The purpose of the present study is, firstly, to develop a new learning approach that engages students in a series of real-life geometrical situations based on the facilities of dynamic geometry software – such an approach could be so-called Story-based Dynamic Geometry – while the second purpose is the attempt to improve students' attitudes towards geometry and geometric proof.

1.2 Research Questions

Guided by the aim of the present study to design a suggested approach, organize its content and examine its effectiveness on improving students' attitudes towards geometry and geometric proof, the following questions will be answered in the following chapters:

1. What are the aims of the suggested approach?

2. What is the content of the suggested approach?

3. What are the learning prerequisites for studying the content?

4. What are the new competencies of every learning activity?

5. What are the learning phases in every activity?

6. What are the students' roles during these phases?

7. What are the teacher's roles during these phases?

8. What are students' attitudes towards geometry and geometric proof?

9. What are students' background and attitudes toward using computers in learning?

10. What is the effectiveness of the suggested approach in improving students' attitudes towards geometry and geometric proof?

11. What are the students' attitudes towards the suggested approach and its content?

Also, in chapter 3 "study methods and procedures" and according to the literature review, questions from one to seven will be answered, while questions from eight to eleven will be answered in Chapter 4 which presents results of the study.

1.3 Research Hypotheses

As speculations upon the answers to questions ten and eleven, two hypotheses were formulated and tested statistically, in addition to an analysis of the most significant items in the questionnaires of attitudes towards geometry and geometric proof, towards using computers and towards the suggested approach:

1. There are significant differences in mean ranks of Egyptian students between pre- and post- administering of attitudes' questionnaire towards geometry and geometric proof.

2. There are significant differences in the mean ranks of Egyptian students between attitudes towards learning using the computer and attitudes towards using the suggested approach.

1.4 Study Treatment's Materials

Story-based dynamic geometry approach was used to engage students in two types of activity. The first was the real-life geometry situations, which were included in the geometric story; this type is a prerequisite for studying the second type, which contained abstracted geometry theorems.

1.5 Study Instruments

Data were collected based on three questionnaires of attitudes towards geometry and geometric proof, towards using computers in learning and towards the suggested approach. In addition an entrance test was given, which was used in order to learn to what extent the students have the required geometry knowledge to study the content of the suggested approach. Further, an achievement test was designed with the purpose of deepen or duplicate the present study to examine students' achievement for the included geometric content in the suggested activities.

1.6 Subjects of the Study

The community for the present study sample comprised the students in the Faculty of Education in Suez Canal University, Ismailia, Egypt and the students in the University of Education Karlsruhe, Baden-Württemberg, Germany. Also, the participants in the present study were selected according to three criteria: the students have a negative attitude towards geometry and geometric proof, the students have the prerequisites to study the content of the suggested approach and, third, which was an ethical issue, the students who take part in the present study experiments actually have an interest in participating. Consequently, the number of participants was 220 students before the experiments (20 Egyptian students and 200 German students), only 18 students (12 Egyptian students and 6 German students) were available for the study experiments, and 13 students (12 Egyptian students and 1 German student) for the post-administering phase. It is obvious that the drop-out rate was extremely higher among German participants than Egyptian, and this might be a result of the freedom regarding whether or not to attend lectures at university, especially when there is no final examination at the end, which was indicated by some of the students and one of their lecturers.

1.7 Limitations of the Study

1. According to the drop-out among German participants, the results of the present study will be limited to the group of Egyptian students in the Faculty of Education, Suez Canal University, Ismailia, Egypt who will be mathematics teachers. Where the results obtained from the second group of German students at University of Education Karlsruhe, in the pretest phase, will give a profile of their background about using dynamic geometry software, in addition to their profile of attitudes towards geometry and geometric proof.

2. Field of the study: Using dynamic geometry software in developing a new organization for mathematical content, and a suggested teaching approach to improve students' attitudes towards Euclidean geometry and geometric proof.

1.8 Importance of the Study

This study is supposed to:

1. Help the prospective teachers (math section in the faculties of education) to change their attitudes towards geometry and geometric proof.

2. Help the prospective teachers (math section in the faculties of education) to overcome some difficulties in understanding geometric theorems, formulating geometric statements by themselves, finding proof idea and scaffolding proof writing.

3. Focus the attention of mathematics teaching staff in faculties of education on the importance and other functions of proof more than it being just a means of confirming the trueness of geometric theorems.

4. Focus the attention of mathematics teaching staff in faculties of education on the importance of educational technology, such as dynamic geometry software, in geometry teaching and learning situations.

5. Focus the attention of the mathematics curricula developers in Egypt on real-life geometry as a launching point to introduce students to geometric concepts and theorems.

6. The study suggests a new way to organize mathematical content in an integrated form that depends not only on the logical structure of the knowledge, but also on a context that is relevant to the real world such as a story that requires an optimal solution in every scene.

1.9 Definitions of Terms

In order to avoid misunderstandings, clear definitions of the main terms are necessary for reading the present study.

- **Attitudes towards Geometry and Geometric Proof** refer to students' beliefs about the importance of geometry as a learning topic and in real life, the importance and functions of proof in geometry, the possibility of doing geometry. It also refers to students' emotions that include subcomponents such as enjoyment of geometry learning, self-confidence in learning situations, learning motivation, frustration in learning situations, fear of geometry learning and anxiety of geometry learning. Both students' beliefs and emotions reflect on how students behave in geometry class, which refers to active participation in geometry class, working with a partner and sharing ideas, observing and expressing conclusions, showing understanding of geometric concepts and theorems, posing relevant questions, and constructing geometric configurations.

- **Dynamic Geometry Software** is defined as one part of an interactive mathematics-learning environment, which allows users to create and control geometric objects such as points, and constructions such as lines. It magnifies geometric construction to see what was difficult to recognize using the traditional geometric tools; also, after one theorem is constructed, students can move one point and see how the construction behaves.

- **Geometric Story** refers to the way that the suggested content was organized in a series of real-life situations that required optimal solutions. Students can acquire new geometric concepts and theorems by themselves through investigating every situation in the story.

1.10 Study Procedures

The present study was conducted over the following steps:

1. Reviewing the related studies and literature
2. Defining the characteristics of the targeted learners
3. Selecting the study sample
4. Stating a preliminary vision of the suggested approach
5. Defining the aims of the suggested approach
6. Choosing the content of the suggested approach
7. Organizing the content of the suggested approach
8. Developing the activities CD-ROM
9. Designing the measurement tools of the study
10. Examining the suitability of the activities and the measurement tools through a pilot experiment and according to expert opinions in the field.
11. Administering treatments and measurement tools in the main study experiments
12. Collecting and analyzing data
13. Concluding results and suggestions.

1.11 Structure of the Dissertation

In order to frame this study coherently within the related studies and literature, the next chapter reviews the related studies and literature. Chapter 3 “study methods and procedures” presents the suggested approach, the development of the CD-ROM, instrumentation, sampling and the procedures of the main study experiments. Chapter 4 presents the results of the study. And finally, Chapter 5 offers a conclusion of the present study, suggestions, and recommendations for future studies.

2 Review of related Literature

The major aim of the present study is to improve students' attitudes towards geometry and geometric proof. Therefore, this study was concerned with developing a suggested approach based on the facilities of dynamic geometry software, in order to engage students actively in real-life geometry situations. Thus, this chapter surveys the related studies and literature over five sections. The first section deals with attitudes, definitions, improvements and measurement tools. The second section sheds light on the status quo of geometry and geometric proof. Furthermore, it reviews the prior studies that were conducted to investigate students' difficulties and their negative attitudes towards geometry and geometric proof in addition to the attempts of overcoming such learning difficulties, using different learning strategies and using new learning technologies. The third section presents some of the areas related to the present study, such as geometric reasoning, thinking levels and the meanings and functions of geometric proof. Section 4 deals with the importance of engaging students in geometric activities, starting with a real-life situation, which could activate the principles of learning theories, such as cognitive and social constructivism. The last section provides a view of the role of dynamic geometry software and its facilities in presenting these real-life geometric situations to overcome students' difficulties, negative emotions and false beliefs about geometry and geometric proof.

2.1 Attitudes: definitions and components

Many studies have been conducted to examine attitudes toward various learning subjects and related variables such as achievement and learning difficulties of these subjects. However, most of these studies aimed at improving students' attitudes from negative to positive feelings. Actually, these studies accepted a "one-dimensional" definition of attitudes, as just students' emotions toward learning subjects and situations; see, for example, in the study by Fan, L., et al. (2005) that aimed at assessing Singapore students' attitudes toward mathematics and mathematics learning, the researchers gave ten examples from the study questionnaire items, which focused entirely on the affective domain. They used expressions such as: *I enjoy*, *I am not* *afraid, have confidence, I like,* and *I am sure.* These expressions showed that the researchers accepted the simple definition of attitudes towards mathematics as just being students' emotions.

In another attempt to investigate more components of attitudes and the relation between them and achievement in Euclidean geometry, Mogari (1999) examined four components of attitudes: enjoyment, motivation, perception of the importance of geometry and freedom from fear of geometry. The results indicate that there were very weak relationships between achievement and each of the four variables. According to the multicomponent definitions of attitudes, the present researcher considers the four variables in the study of Mogarti as only two components of attitude while enjoyment, motivation and freedom from fear of geometry could be categorized as emotions, and perception of the importance of geometry as merely one element in the beliefs about Euclidean geometry.

A meta-analysis study was conducted by Ma and Kishor (1997) aimed at reviewing 113 studies that had investigated the relationship between attitudes towards and achievement in mathematics. Most of studies' results showed a week relationship between the two variables. This was a result of measuring only one of the components of attitude to examine whether there is a change from negative emotion towards mathematics to positive. This was consistent with the results of the previous two studies which showed that the restriction in the structure of the questionnaires' components could be the reason for the weak relationships found between achievement and attitudes, while both studies were dependent on just one component of attitude which could be classified as "emotions".

In comparison, Pickens (2005, p.44) went beyond this simple view of attitudes, and clarified that attitudes might help us to define how students can see the different situations, how they behave in these situations and reflect their feelings, thoughts, and actions. Hence, attitudes give a view about the structure of the internal beliefs, thoughts, and how these two components affect students' behavior in different situations. Di Martino and Zan (2003) agreed with this "multicomponent" definition, which included the affective, cognitive components and behavioral tendency.

Zan and Di Martino (2007) in a more recent publication came up with further explanations and articulated that in most questionnaires of attitudes, an answer can be characterized as a positive answer, which might refer to different meanings. These meanings varied depending on what the word positive refers to: emotions, beliefs or behaviors. So, when the word positive refers to emotions such as feeling anxious when doing mathematics, it is seen as negative; in contrast, pleasure will be seen as positive. When the word positive refers to beliefs, it generally deals with shared meanings and ideas. While successes in learning contexts usually identified with high achievement reflect the meaning of the word positive when it refers to positive behavior.

Gómez-Chacón and Haines (2008) clarified the meaning of positive or negative attitude according to the relationship between the previous components. In this case, the attitude of a student who likes mathematics is defined as “negative” if these positive “emotions” are associated with false beliefs about mathematics, for example, as a set of rules to be memorized. The present researcher wants to emphasize this indication, while the “success and high achievement” of positive emotions and positive behaviors could be the outcome of learning and teaching methods that emphasize rote learning, which reduces the role of mathematics to just memorizing theorems and recall them in the final exam. In this case, the students might be expected to have positive emotions, positive behavior “participating in the classroom to memorize the information, and high achievement in the final examinations which required just recalling such concept definitions and theorems and even proofs”, but the overall outcome was considered to show negative attitudes because both positive emotions and behavior depended on false beliefs about the function of mathematics. This shows the interrelated relationships between these components, so that beliefs affect emotions and both beliefs and emotions affect behavior (Pickens, 2005, p.44). Goldin, Rösken and Törner (2009) stated that there is a need for research into the roles that beliefs might play in opening up possibilities of deeper mathematical understandings and evolving students’ achievements. Thus, the present study accepted the three-dimensional definition of attitudes which is as follows: the first dimension is emotions that include more sub-variables such as enjoyment, confidence, motivation, frustration and fear of geometry and geometric proof.

The second dimension is beliefs that include students' thoughts and perceptions about the importance of geometric concepts and theorems in classroom context, in real-world situations, and the importance and functions of geometric proof. Both the previous dimensions form students' behaviors as the readiness to participate in a geometry class discussion, showing understanding of geometric theorems and concepts and sharing ideas and collaborate with class partner.

2.1.1 Acquiring and Improving Attitudes

Leading from the previous review, when a student has the ability to achieve geometrical concepts and theorems, this does not mean he or she will have the right beliefs of the importance and the relevancy of this knowledge. In other words, competencies mean the ability to achieve or to do geometry, while attitudes towards geometry reflect the motives behind the willingness to do it. As attitudes include a cognitive component – beliefs – these could be changed by different ways and situations in a student's life.

Lamberth (1980) indicated three ways in which attitudes are acquired: classical conditioning, instrumental conditioning and imitation. Classical conditioning can occur through believing in the experiences of other people. In view of that, it is not needed to be involved in the processes that formed experiences of those people. In other words, the student can obtain the desired beliefs by observation. Furthermore, behaviors are part of the components attitudes, according to the multi-dimensional definition, so students might imitate such related behaviors to learning subjects mainly from the teacher (Freedman J.et al, 1976). Pickens (2005, p.48) agreed with this view and concluded that attitude formation is a result of learning, imitating others, and conclusions of direct experiences with people and situations.

In fact, there is a common belief about the difficulty of changing attitudes that differs in regard to the length of the required time for changing or improving (Bootzin, 1983). In addition, the present researcher concludes that there is another reason for such a common view. Trying to change or improve attitudes according to the simple definition of attitudes as just students' emotions makes it a difficult task to accomplish because this view does not consider the cognitive background that led to the students' emotions.

In one model of changing attitudes, Carl Hovland clarified that it is possible to change attitudes without the need for long treatments; the model showed that attitudes can be changed in a short time of merely one class session. Hovland based his view on considering the process of changing attitudes as a communicational situation like that which occurs in convincing customers to buy something (e.g., an advertising process). He posed a multifaceted question that contains some variables in the situation of changing someone's attitudes: Who says, what, to whom, how and with what effect? Hovland was concerned with answering five branched questions: What is the message in such situation? Who is the sender of this message? Who is the recipient? What is the channel used to send the message? What is the effect that should be gained at the end of such a situation? (Hovland, C., 1959)

Accordingly, the present researcher wants to reflect on this view, so that in every learning situation that includes content "the message" relevant to students' needs, they will be able to recognize and appreciate its importance, while the teacher "the sender" can play a vital part in engaging students in learning activities actively, and enabling them to participate in formulating the learning situation and its outcomes so that they go beyond just the passive role of absorbing knowledge. The answer to the third part of the question posed by Hovland showed the importance of considering the characteristics of students – "our customers" – while the fourth part was concerned with choosing the appropriate learning environment – "the channel of communication" – to use in sharing ideas in the learning situation, so as to change students' attitudes towards a learning subject and to convince them of the importance of studying it. Pickens (2005, p.50) mentioned that to change someone's attitude, not only his emotions need to be addressed, but also the cognitive components of such an attitude. For instance, to avoid participating in an exercise program someone might say "*I do not have enough time*" or "*I do not want to risk it*", and so providing new information in this situation would be one method of changing their attitude.

The present study accepts this view of the possibility of changing students' attitudes by considering their cognitive background, their characteristics and their beliefs in order to change their negative emotions, which will hopefully evolve their behavior and especially their participation in the learning situations.

Khalaf (2002) interpreted the term communication as the process of acquiring or transforming thoughts, knowledge, skills and attitudes with the use of several channels such as verbal or written words, symbols, and figures. This is consistent with the view of attitudes as anything that could be learned and acquired. Related to that, this chapter will later investigate the importance of activating the principles of constructivism theory and engaging students in social context.

However, the last part of Hovland's question remains, which is concerned with examining the positive effect that should be gained at the end of such situations, which shows the need for designing measurement tools to investigate whether both learning content (e.g., the message) and teaching methods (e.g., the communication channel) were effective or not.

2.1.2 Measuring Attitudes

Previous studies used many different types of attitudes scales, the most common method being the Likert scale. Some of these studies used their own methods to measure attitudes, such as the study by Gomez-Chacon (2000) that aimed at establishing and describing the significant relations between cognition and emotions in mathematics class, in addition to exploring the origin of these emotions and reactions, and indicating the changes. The study was restricted to measuring just one dimension of students' attitudes, namely their emotions. It used a new technique which centered on drawing a specific sign that enabled students to express their feeling while participating in solving mathematical problems in class. These signs included: to express curiosity, for being cheerful, for feeling despair, for being calm, for being in a hurry, for boredom, for a brain teaser, for liking something, = for indifference, for amusement, for feeling mentally blocked and for expressing confusion and bewilderment. The results showed that the most frequented emotions were calmness, cheerfulness, and being mentally blocked. Furthermore, the results indicated the effects of students' beliefs on their emotional responses during problem-solving situations.

In fact, despite considering the above method as an innovative attempt to measure students' attitudes, it seems that this method like others such as observational and self-report methods are not adequate for measuring other components of attitudes. A component such as students' thoughts and beliefs needs more detailed sentences and perhaps open-response items to enable students to express their previous experiences, which have played a part in forming those attitudes about geometry and geometric proof. The following methods are considered as the most common in the previous studies of measuring and investigating attitudes:

2.1.2.1 Thurstone method:

The participants have to respond to the items of this scale, which ranges between two possible responses: "true or false" or "agree or disagree". With this method, many of items that are related to the aim of the scale need to be formulated, where the next and the main step is submitting the items to a number of specialists in the field to decide to what extent every item is negative or positive and to rate each item regarding its clarity and the suitability. The experts were asked to respond by giving a score for every item ranging from 1 "highly negative" to 11 "highly positive", and 6 for those items where there was hesitation to agree or disagree. The mean rank of all ratings produces the scale value. After that the scale was administered on a number of students to calculate its reliability and to choose a number of items that would form the final attitude scale.

One of the disadvantages of this method is that the judges may not be qualified or interested in the content of the items; furthermore, the judges can differ in their decisions. Another disadvantage is the complexity and the long length of time required to prepare such a scale where the results are not guaranteed (Dwyer, E., 1993).

2.1.2.2 Likert method:

The Likert scale is the most popular method for measuring attitudes, which came to close the disadvantages of Thurstone method. Also, Likert originally used five responses, which ranged from strongly agree (5) to strongly disagree (1). Modification of the number of response alternatives is acceptable and to calculate the overall attitude of a participant, all of his responses for the entire scale's item should be added, taking into consideration the negative items that must be reversed if they exist. It is desirable to formulate more items than are planned to be used, as many of these items could be found to be inappropriate for the target of the scale. Thus, the statistical analysis for all items and their correlation to the whole scale is needed to be calculated by piloting the scale with a group of participants in order to decide which items are related to the whole summation of the scale and should therefore remain, and which items have a poor correlation with the whole scale and should be removed. This might also affect the reliability of the scale as a result, but in both Thurstone and Likert scales, reliability tends to increase with the number of items which may also increase the time taken to completely answer the scale which may reduce the motivation of participants to complete it or to respond seriously to every item. So, there is no exact number of scale items, but the appropriate number is between 20 and 30 items (Dwyer, E., 1993; Pickens, 2005).

It is important to mention another method for analyzing the items statistically, which is a statistical technic called "item-total reliability analysis". This technique depends on calculating the reliability for the scale including all the formulated items, and calculates the reliability again but after deleting every item. This method shows if one item is deleted whether the reliability tends to increase; if so, it is recommended to delete this item from the final version of the scale because its existence affects the total reliability, and if the reliability tends to decrease after deleting an item, it is recommended to keep this item in the final version of the scale.

The Likert technique was used in the present study with a modification to the scale of responses on every item to fit the aim of the designed scale. While in the original scale of Likert the intermediate response was undecided and takes the value 3, the present researcher considered this a disadvantage and the scale should be modified to the value 0 with the meaning "I do not know!" This would mean that the participant might not have had the experience that the item presents it or that the participant hesitated to respond. Both of these should be considered statistically as a passive response if it takes a value of 3. In addition, an open-response item was used to enable students to report further thoughts and feelings about geometry and geometric proof.

One of the most important issues that should be considered in preparing measurement tools is validity. There are many methods to verify the validity of a scale or a test. Validity is concerned with determining to what extent a test or scale fits with what it was aimed and planned to measure.

The simplest way to calculate the statistical coefficient of validity, known as experimental validity, is by finding the square root of the reliability coefficient (Angoff, 1988, p. 20). Dwyer (1993) mentioned other types of validity such as content validity, construct validity, face validity and curricular validity. Face validity answers a simple question: “Does the test appear to measure what it aimed to measure?” (Dwyer, 1993, Pp. 27) while the content validity, according to the American Psychological Association definition, is the degree to which items are representative of the traits being measured in the test. The construct validity is concerned with expressing the relationships in the content structure; however, both content validity and construct validity cannot be indicated in one coefficient like experimental validity. The curricular validity, as Cronbach (1960) mentioned, is the type that concerns determining whether the test reflects the content and the instructional goals or not.

2.2 The Status Quo of Geometry and Geometric Proof

Obviously, there is an agreement on the description of the traditional school mathematics. Civil (2002) stated some characterizations by mentioning the excessive dependency on paper-and-pencil computations with little meaning, clearly formulated problems following prescribed algorithms, and a focus on symbolic manipulation deprived of meaning. Christou et al. (2004, p. 216) stated that the traditional teaching emphasized that a mathematical statement is true if it can be proved, while the function of proof has been seen almost and exclusively in terms of the verification of the correctness. At the secondary level, Kemeny (2003) referred to the traditional Euclidean geometry curriculum that revolves around deductive proof procedures; this has been criticized because it separates geometry from its empirical, inductive foundation. Critics referred to the typical lack of student appreciation for the subject – often accompanied by low achievement.

In a study of sixteen high school grade teachers, the results referred to the false beliefs among many of the teachers about the role of proof as just an argument that demonstrates the truth of a statement (Knuth, 2002). However, one teacher's response was particularly interesting. When asked to define a proof, she answered: "*A convincing argument showing that something that is said to be true is actually true.*" The most positive result came from the responses of twelve teachers who expressed their view about proof is an outcome of social interaction. So as some of them stated that "*proofs are a means to communicate and convince others of one's claim*", this point of view again reflects a common conception about proof as a means of verification.

In addition, teachers often find that the most difficult part of the process of writing a deductive argument is finding the basic logic of the argument and the required information. Most students do not understand how they have reached the concluding proof once they have completed the problem. Gfeller (2005) referred to one of the most common students' beliefs with the concept of proof being non-deductive arguments that are enough to establish a proof. In addition, he stated that students may be able to produce proofs without actually understanding them.

In the following there is a review of the prior studies that investigated and attempted to overcome the difficulties and negative attitudes towards geometry and geometric proof.

2.2.1 Studies investigated students' attitudes and difficulties

Many studies have been conducted to investigate students' attitudes towards mathematics. In this section, the present researcher aimed at surveying those that were concerned with investigating students' difficulties and attitudes towards geometry and geometric proof.

However, before presenting the procedures and the results of these studies, the researcher wants to state that some of these studies investigate attitudes in terms of just being emotional responses, while the multi-dimensional definition includes many other components in attitudes as presented in a previous part of the chapter.

Almeida (2000) conducted a quantitative study of a large sample of UK undergraduate students: 351 from year 1, 117 from year 2, and 5 from year 3. The researcher used a sixteen-item Likert scale proof questionnaire. The quantitative results show that the statement 7 "*I cannot see the point of doing proofs: all the results in the course have already been proved beyond doubt by famous mathematicians*" has the following mean responses: 2.27 for year-1 students, 2.14 for year-2 students, and 2.2 for year-3 students.

These values in comparison to the ideal response reveal that this item is the most negative item in the questionnaire and can be interpreted as evidence that students cannot see another function of proof other than just a test of the correctness of a statement. Students see that it is sufficient for the theorems to be true just because they are the work of famous mathematicians. The means of the responses to statement 9 "*if a result in mathematics is obviously true then there is no point in proving it*" were 2.59 for year-1 students, 2.45 for year-2 students and 2.8 for year-3 students.

This result confirms the interpretation of the results of statement 7. Both statement 10 "*I like doing proofs in mathematics*" and statement 11 "*I am confident in my ability to prove results for myself*" had responses ranging between disagree and no opinion, which shows the dislike and lack of self-confidence in proof-learning situations.

In the University of Stockholm, Nordstroem (2003) reported the results of a study designed to investigate the attitudes towards proofs of 100 Swedish university entrants.

In comparison to the results of Almeida's questionnaire, Nordstroem concluded that the students have positive attitudes towards proofs and stated that the results must be carefully interpreted, as the questionnaire was distributed to the students at a time when they were about to enter their studies at the mathematics department, where they had freely chosen to study.

At this point, an important factor should also be considered, since Almeida's questionnaire ranged in five responses from strongly disagree to strongly agree, in Nordstroem's questionnaire students responded to only one of two choices: agree or disagree. This could increase the error of measurement and may motivate students to think about the ideal response that should be given.

According to the multidimensional definition of attitudes, the researcher in the present study considers the students' view of the proofs of functions as one of these dimensions. Another study conducted by Gfeller and Niss (2005) over four months with 15 tenth grade students in the style of participant-as-observer, aimed to investigate the students' views about the purpose of geometric proofs. The data were collected through direct observations while the students worked in small groups in addition to a post-instruction questionnaire.

Descriptions of the students' views of geometric proof were classified into five categories: verification, explanation, communication, systematization, and discovery. The results show that students expressed limited views of the purposes of geometric proof, referring mainly to verification.

It seems that not only the students' attitudes towards proof were limited to the function of verifying the rightness of a geometric statement. A study by Knuth (2002) examined 16 in-service secondary school mathematics teachers' conceptions of proof using semi-structured interviews. The launching point of the study was the argumentation that secondary school mathematics teachers should provide all students with rich opportunities and experiences with proof that reflect the roles of proof and depend on the teachers' conceptions of proof.

The most significant response was that only three teachers expressed that one of the functions of proof is giving an answer as to why a conclusion is true, whereas all teachers indicated that the function of proof is to establish the truth of conclusions. Knuth concluded that there was no supporting evidence that teachers view the promotion of understanding or insight as a role of proof in mathematics and reported two examples from the teachers' responses: "*I think it means to show logically that a certain statement or certain conjecture is true using theorems, logic, and going step by step.*" Moreover, that proof is "*a convincing argument showing that something that is said to be true is actually true.*" The teachers' responses also show that twelve teachers expressed the view that proof is a product of social interaction. Only eight teachers expressed the view that proof plays an important role in creating mathematical knowledge.

In Zimbabwe, a study of thirty-four secondary school mathematics teachers' attitudes towards proof was conducted by Buzuzi and Nyaumwe (2007). Participants responded on a Likert questionnaire during the first 45 minutes of a day-long in-service teacher training workshop, besides interviews with six teachers. This enabled them to investigate their proof teaching' methods, suitability of students' level to perform proofs, and students' attitudes. Descriptive statistics were used to gain insights into the teachers' attitudes towards proof in secondary school mathematics curriculum. The results showed that the teachers agreed, but not strongly, on items that belong to the positive attitudes. The results of the interviews showed that the teachers expressed the belief that proofs can verify the viability of mathematical results and can help train students to reason logically. Buzuzi and Nyaumwe concluded that the following responses illustrated teachers' attitudes: "*proofs are important in order to verify that a mathematical result is true*" and "*proofs train learners to think logically and present their arguments sequentially.*" Moreover, the results of the interviews showed that some of the teachers' attitudes towards mathematical teaching were informed by formalist conceptions rather than social constructivist theories that encourage students' construction of mathematical results and proving them.

Many other studies confirmed the relationship between attitudes and achievement, where the frustrations in mathematics class may contribute to learning difficulties and continuing in a vicious circle leads to more negative attitudes and false beliefs.

Eleftherios and Theodosios (2007) focused in their study on the relationship between students' attitudes about studying mathematics and their ability to understand mathematical proofs. In addition to investigate, whether there are any differences in students' beliefs according to gender or social status. The data collected by a questionnaire administered to 1645 students of the 10th, 11th and 12th grade in Athens, Greece. The questionnaire consisted of 28 statements, 10 of which were concerned with beliefs and 14 with attitudes about mathematics. The 25th question concerned students' performance in mathematics at school in the previous year. The 26th, 27th and 28th item was called math test and aimed to measure the students' ability to understand mathematical proofs. The results clarified that the attitudes towards mathematics correlates positively with the ability of reflection and understanding proofs.

Moore (1994) conducted a study to examine the cognitive difficulties that 26 students in the University of Georgia experience while learning to do formal mathematical proofs during a course including mathematical logic and methods of proof. The data were collected primarily through non-participant observation of class each day, interviews with the professor and the students, and tutorial sessions with the students outside of class. The results of the study revealed the following sources of students' difficulties in doing proofs:

1. The students did not know how to use definitions to obtain the overall structure of proofs.
2. The students were unable to understand and use mathematical language and notation.
3. The students did not know how to begin proofs.
4. The students had little intuitive understanding of the concepts.

In addition, the students' perceptions of mathematics and proof influenced their proof-writing performance and were sometimes a hindrance to their success.

These results are consistent with another study conducted in Egypt by AbdelGawad (1994), which was concerned with developing geometry proof teaching in preparatory school. The findings revealed that:

1. The students did not know how to complete the proof.
2. Some of students do not note the reasons of every conclusion in the proof.
3. The students cannot use the appropriate language to express the proof ideas.
4. The students cannot organize the conclusions in logical sequences.

In Spain, Recio and Godino (2001) aimed to investigate mathematical proof schemes of students starting their studies at the University of Cordoba and the relationship between these schemes to the meanings of mathematical proof in daily life. A questionnaire was given to the 622 students who studied mathematics in different faculties and who had started at the University of Cordoba just a few days previously.

The results revealed the low level of writing deductive proofs in addition to the difficulties in distinguishing the intuitive argumentation they use in their daily life from the deductive reasoning required in the mathematics classroom.

2.2.2 Studies aimed at overcoming students' difficulties and attitudes

This section is concerned with studies and projects over the past few years, which used different treatments and approaches such as drama-based and daily-life mathematics, cooperative learning, and computer assisted learning in order to overcoming students' difficulties and negative attitudes in geometry and geometric proof.

Duatepe-Paksu and Ubuz (2009) investigated the effects of drama-based geometry instruction compared with traditional teaching on students' achievement, attitudes, and geometric thinking levels. The study sample was 102 seventh-grade students in a public elementary school in Ankara, Turkey. Two achievement tests consisting of 49 items were used to determine the achievements of some geometric concepts such as angles, polygons, circles, and cylinder. In addition, interviews were used with students in order to gather details about their attitudes towards geometry and the drama-based instruction method.

The quantitative results showed that drama-based instruction had significant effects on students' achievement, attitudes, and their geometric thinking levels. The qualitative analysis of students' responses in the interviews showed that:

1. The drama-based examples affected students' learning positively.
2. The examples made students realize the connections between life and geometry with the help of a daily-life context.
3. Students were able to acquire knowledge because it was given in a meaningful context.
4. The drama-based instruction enhanced communication and a collaborative learning environment.

It seems that the previous study was not the first study to use daily-life contexts. At the Pittsburgh Teachers Institute, a report by Casciato (2003) describes the UCSMP project, which was founded in 1983 aiming at giving students the opportunity to explore geometry through the everyday mathematics curriculum. The project emphasized the importance of problem solving, real-world applications, use of calculators and computers. Students in each year revisit concepts that were previously taught, in order to provide a comfort level with the topic and make it less distressing. At the end of the report, Casciato recommended that it is very important to enable students to work together with class partners or in groups that encourage their attitudes to question. Also, there is an agreement between these results and recommendations with another study conducted by Nichols (1996) at the University of Oklahoma, which aimed at examining the effects of cooperative learning on student achievement and motivation in a high school geometry class. In this study, eighty students responded on an 83-item Likert questionnaire and an achievement test, in order to measure their geometry achievement, goal orientation, self-efficacy, motivation toward geometry, and cognitive processing. Participants in this study were divided into two groups, which differed in terms of the style of instruction. Both the control group and cooperative learning treatment group were provided with the same material of plane geometry. The results of the study reported that cooperative group learning increased students' achievement and motivation to learn. The results also positively indicated the impact of cooperative learning instruction on the use of deep cognitive processing, the adoption of learning goals, and self-efficacy.

Another study conducted by El Sayed (2000) aimed at identifying the effectiveness of problem-posing strategies on prospective mathematics teachers' problem-solving performance in the Sultanate of Oman. Fifty student-teachers participated in the study over the period extending from November 12, 2000 to December 17, 2000. They were divided into two groups, an experimental group and a control group; each group consisted of 25 student-teachers. All students underwent an achievement test consisting of eight open-ended problems relevant to everyday life situations like shopping and vacationing problems. The results showed a significant improvement in student-teachers problem-solving and problem-posing performance where they had opportunities to discuss each step with greater emphasis on the use of problem-posing strategies.

In Egypt, a study conducted by Refaat (2001) aimed at developing preparatory school students' mathematic proof skills and geometry achievement using module-based instruction. The treatment was concerned with developing students' ability to write a two-column formal proof and to use simple language to write the proof. The results showed that, compared to traditional geometry instruction, module-based instruction was effective in developing students' skills to write geometric proof in addition to overcoming their geometry-learning difficulties. Another more recent study conducted by Refaat (2005) aimed at overcoming students' learning difficulties and reducing their anxiety regarding geometry. The sample was also from the preparatory school students in Ismailia and Port Said, but using a different treatment using a multimedia computer program and based on principles of constructivism.

Refaat reported the geometry teaching methods in Egypt whereas all schools in all governors have the same mathematics textbook in every grade, although geometry teaching and learning still depends on the traditional methods, which do not consider the role of students in constructing knowledge. Several measurement tools were used in this study, varying between semi-structured interviews, a geometry anxiety test, and van Hiele geometric thinking levels test translated into Arabic. The results of the study revealed that the treatment using the multimedia computer program based on constructivism reduced the students' anxiety about geometry-learning situations. Additionally, the treatment was effective at overcoming students' learning difficulties in geometry topics.

In Australia, Vale and Leder (2004) aimed to examine students' views of computer-based mathematics instruction in addition to investigating whether or not there is a difference in these views due to gender. Forty-nine students (17 girls and 32 boys) from grade 8 and grade 9 in a secondary school participated in the study. The ratio of girls to boys was almost 2:1 and they were taught in Victorian classrooms. The students had been working on laptops with Excel in the previous semester and were using geometer's Sketchpad in the study experiment. They completed guided investigations in computer-based lessons such as investigating the exterior angles of a polygon. The data sources were questionnaires, classrooms observations, and interviews with four students from each grade. The results of the study showed that girls viewed the computer-based lessons less favorably than boys did, whereas boys believed that computers are more relevant to doing mathematics. Girls were more concerned about the computer as a facilitator for learning than boys. The results reflected a significant relationship between attitudes of students towards using computers in mathematics learning and their general views of computers. The present researcher assumes that it is more essential to know teachers' attitudes towards computer use those of students. While teachers expected to be the best one who can transfer and convince students with the importance of using such new technologies as well.

Norton et al. (2000) began a study exploring secondary mathematics teachers' reasons for not using computers in their teaching by stating that: "*Despite the availability of hardware and software, the mathematics staff in a technology-rich secondary school rarely used computers in their teaching*" The aim of Norton's study was, firstly, to investigate the relations between teachers' beliefs about teaching mathematics and their practices and their attitudes toward using computers in their mathematics teaching and, secondly, to examine the staff discourse that facilitates or hinders the use of computers. The study was carried out with 650 students in seven laboratories; each one had 25-30 computers. Thus, every four students were allowed to work on one computer. The data sources of the study were interviews with teachers, observing and audiotaping several lessons, and discussions about these lessons. The results of the study revealed that the teachers who were content-focused, time-factor anxious and teaching-centered had a restricted view of using computers in teaching and learning. In contrast, teachers with views of teaching consistent with social constructivism learning theory had a broader image of using computer in teaching and learning. The results also indicated individual teachers' resistance related to their beliefs about mathematics learning and examinations.

In another study of the factors related to teachers' use of technology in secondary school geometry instruction, Coffland & Strickland (2004) used a telephone survey consisting of both closed and open questions with 136 teachers from 52 different schools in the state of Idaho in the U.S.A. The survey packet was sent to all geometry teachers in schools; then all such teachers were asked to participate in and respond to the survey. The purpose was to discover the relationship between teachers' awareness of using technology in geometry teaching, computer use in secondary school geometry class, and the number of geometry lessons taught. The results were obviously significant, with a negative relationship were found between teachers' computer use and the number of geometry sections taught. In addition, there was a positive correlation between teachers' attitudes towards computer use in geometry instruction, type of teachers' training, teachers' awareness of using technology in teaching, teachers' experience and the availability of a computer laboratory.

2.3 Reasoning in Geometry

This section sheds light on some concepts related to students' learning difficulties and negative attitudes, such as the geometry thinking levels, the meaning and the functions of geometric proof. However, before investigating the above concepts, it is very important to clarify what the kinds of reasoning in geometry are and what the main processes of geometric reasoning are.

There are two basic kinds of reasoning: inductive reasoning, which is used to classify geometric configurations visually and/or mathematically, so as to enable student to make intelligent guesses based on those geometric configurations. Inductive reasoning includes cause and effect relationships, it is also depends to some extent on students' previous experiences. The second kind is deductive reasoning, which explains the reasons behind the trueness of geometric theorems.

However, in geometry students can learn through measuring distances, angles and making conjectures about the geometric constructions and relationships. In other words, inductive reasoning is the work through experimentation and observation of visual and/or numerical relationships that might lead to making conjecture. Deductive reasoning explains the meaning of the term theorem; whereas the mathematical truth can only be established if someone can prove that the trueness of the conjecture does not depend on limited observations or special cases. Kemeny (2003) clarified that the inductive line of reasoning that begins with empirical investigation and the observation of regularities is usually followed by tangible discovery. It continues with making conjectures based on the observed regularities, and then testing them on multiple examples. Attempts at explaining and generalizing the observed relationship with the help of proof come only after the long process of empirical exploration.

In a study of adult students' reasoning in geometry, Bjuland (2002, p. 2) defined several reasoning processes instead of focusing on the process of proving. These processes are sense-making, conjecturing, convincing, reflecting, generalizing and justifying. The results of the study also suggested that both visualizing and questioning besides collaborative context, play important roles making progress in stimulating students with limited mathematical background to improve their reasoning and problem-solving skills.

2.3.1 Geometry Thinking Levels

One of the best known theories in mathematics research was developed by Pierre and Dina van Hiele – husband and wife – which claims that when students learn geometry they face difficulties in progressing from one of five sequential and hierarchical levels (recognition, analysis, ordering, deduction and rigor) of geometrical thinking to a higher one if they do not have appropriate experience at the lower level. The relevancy of this theory to the present study can be summarized in the assumption that skipping one level in the learning and teaching processes might lead students to enter a vicious circle of negative attitudes and difficulties in geometry class. De Villiers (2004) described the first four van Hiele levels as follows:

Level 1: recognizing the geometric constructions visually by their appearance without consideration for the geometric properties

Level 2: identifying, describing and analyzing the geometric constructions by the properties for each construction separately

Level 3: recognizing different constructions by seeing the relationships between their properties and writing short sequences of deduction

Level 4: developing longer statements of deduction, understanding the significance of proofs and using the sufficient conditions (De Villiers, M., 2004, p.706).

Level 5: comparing different deductive systems and exploring different geometries depending on various postulates' systems (Pegg and Davey, 1998).

The levels of recognition, analysis and deduction were revised by Hoffer (1981, p.15) in the form of a two-dimensional matrix. The second dimension in this matrix contained three main geometric skills: visual, descriptive and logical skill.

Table 1: Hoffer's matrix of geometric thinking levels and geometric skills

Levels / Skills	Recognition	Analysis	Deduction
Visual	Recognize geometric constructions by its picture without knowing the constructions properties	Recognize the relationship between different kinds of geometric constructions	Uses information about a geometric construction to deduce more information
Descriptive	Naming a geometric construction and explain statements that describe geometric construction	Describes the relationships between geometric constructions. Defines geometric concepts clearly	Understand the difference between the definition, postulate and theorem
Logical	Understand the meaning of construction reservation in different situation	Uses the properties of geometric constructions to identify the subset relations	Uses logic to prove and being able to deduce new knowledge from given facts

The previous matrix played a vital part in developing the suggested approach of the present study, in order to make a smooth transition to the level of logical deduction or that so-called proving level, which could overcome the frustration that students face by introducing them directly to such a higher level, especially students who have difficulties in learning geometry and geometric proof.

Miyazaki (2000) suggested that

> *"Verbalization can be a clue to recognize reasoning between viewpoints as a proposition, because verbalization seems to be helpful in order to reflect or objectify one's thinking, and as a result, a proposition becomes obvious in students' minds."*

In view of that, the present researcher wants to emphasize the teacher's role in stimulating students' participation by encouraging them to express their thoughts in their own words, and then to unify their conclusions (e.g., participations) in appropriate mathematical language. As students in mathematics class have different thinking levels, they are expected to express their conclusions in different formulations. Also, one student's conclusion might be seen in different ways, and might be very difficult to understand for students in other levels. This feature of geometry thinking levels could be a source of negative attitudes towards geometry and towards higher levels of geometric thinking (e.g., towards geometric proof), especially when the teacher's feedback is limited to enhancing students' favorable participation. Ross (1998) stated that mathematical terms serve as a vehicle of communication in the classroom, so students should learn them to communicate their thoughts in a commonly accepted language. At this instant, it is important to mention the importance of engaging students in a social learning context; this might be the key to developing their mathematical language, which is required to expressing conclusions, especially in the higher levels of geometric thinking that includes the ability to produce justifications. Boero (1999) suggested sequenced phases toward proof construction; he also presented these phases as one learning context which cannot be separated into more than one session; while in the mathematicians' work, these phases are usually interconnected in non-linear ways.

This learning context is summarized in the following six phases:

1. Encouraging students to produce conjectures.

2. Sharing ideas and reformulating conclusions.

3. Exploring and visualizing possible relations.

4. Presenting arguments under appropriate guidance.
5. Organizing arguments into a proof in acceptable terms.
6. Formulating the formal proof.

The present study benefits from this view by introducing students to geometric tasks starting with visual recognition in an attempt to reach the level of deducing new mathematical knowledge. It also reflects on the results of a study conducted by Senk (1989), which investigated relationships between van Hiele levels, achievement in geometry and particularly in writing geometric proof. The results showed that high school students' ability to write geometric proof is positively related to van Hiele levels of geometric thought and to the non-proof geometry achievement. The results also reflected the chance of mastering proof writing was at least 50% for students who were able to recognize geometric figures and to describe their properties, in other words students who reached the second level of geometric thinking, while students who could reason from definitions (e.g., level 3) have an even greater chance of mastering proof writing. Senk concluded the critical nature of the second level, and also that there is no specific van Hiele level that ensures success in proof writing. And consistent with van Hiele, Senk also concluded the difficulty of writing proof for students below the third level of geometric thinking, so the only way for them is memorization.

At this point, we are invited to think about the consequences of students' attitudes towards geometry and geometric proof, whenever they were involved directly in the level of formal deduction, even at university level.

2.3.2 Meanings and Functions of Proof

Ross (1998) argued that before discussing the reasons why proofs should be taught and how students can learn to do meaningful mathematical proofs, it is important to consider the meanings of proof and its different functions. Also, Tall (1998) under the title "*The transition to advanced mathematical thinking*" stated that mathematical proof as a human activity requires not only the knowing of definitions and understanding the logical processes, but also insight into 'how' and 'why' it works.

The word ‘why’ could be the most frequently used word in the early real-life world of every child. Proofs are often the answer to the same question in mathematics class, particularly in geometry, that can deepen students’ understanding not only of geometric concepts and theorems but also of the real-world around them.

In fact, proof has a primary purpose in geometry class, which is to evolve students’ abilities to reason logically. However, the teacher should differentiate between the main goal, which students should experience several processes to achieve it, and the first task, which is one of those processes. Thus, it is not suggested to begin the learning journey by introducing students to a high-level aim of what is to be achieved at the end. This could lead them to the undesirable outcomes of learning difficulties and negative attitudes.

Hersh (1993, p.391) presented three meanings of proof: the first is the linguistic meaning that comes from the Latin probare, and is cognate to ‘probe’, ‘probation’, probable’, ‘and probity’. The two mathematical meanings are seen as an argument that convinces qualified judges, and as a sequence of transformations of formal sentences, carried out according to the rules of the predicate calculus. Hersh has further stated that real-life proofs are to some degree informal. When around 100 university-level students responded to questions about proofs and proving, nearly half stated that they had teachers who often proved statements as a part of class learning, but only about 30 percent of them said that they had had the possibility to exercise proving themselves (Nordstrom, 2003).

Schalkwijk (2000) stated:

> “*If learning is understood as the undertaking of learning activities that result in stable changes in knowledge, beliefs, and skills, then teaching must enable students to perform those learning activities to lead to the desired outcomes.*”

In view of that, teachers should create innovative activities that bring proof down to a learning level that students can work with.

In his article, "*Proving is convincing and explaining*" Hersh (1993) told us that the role of proof in the classroom can be completely different to its role for mathematicians. He defined the role of proof in mathematical research as a way to convince, whereas in the classroom, convincing is no problem. "*Students are all too easily convinced,*" he says.

Hanna (2000, p. 9) concluded some further functions of proof more than verification such as:

1. Providing insight into why the theorem is true
2. Organizing various conclusions into a deductive system.
3. Exploring or discovering new conclusions.
4. Collaborating and sharing mathematical ideas.
5. Constructing and formulating geometric statements.
6. Incorporating and seeing facts in a new framework and viewpoint.

De Villiers (2004) referred to another function of proof, which could play the role of intellectual challenge.

2.4 Why Real-life Situations?

The National Council of Teachers of Mathematics in the U.S.A. referred to the possibility of using geometric ideas to represent and solve problems in the real world, in addition to the importance of studying measurement because of its relevancy to so many real-world contexts (NCTM, 2009).

Many studies attempted to make this connection between mathematics and the real world using several approaches and several treatments. For instance, Duatepe-Paksu (2009) conducted a study to investigate the effects of drama-based instruction on students' geometry achievement, geometric thinking levels and attitudes toward mathematics and geometry. The results showed, in comparison to the traditional teaching, that drama-based instruction made learning easier and provided students with more opportunities to experience geometric concepts and problems in a social context, collaborative learning environment and motivations to learn the included new geometric knowledge. Furthermore, Duatepe-Paksu stated that in terms of Vygotsky's ideas, learning is shaped both by internal processes and social interaction. Drama provides active communication among students and between students and teachers.

Yet, it seems that is not enough to use everyday mathematics simply as a source of motivation in the beginning of the geometry class. Civil (2002) argued that by going deeper into mathematizing everyday situations, students may be losing what made them motivated in the first place. That was one of the motives of the present study. Braiding more than one everyday geometric situation in a geometric story could maintain students' motivation to find optimal solutions or explanations for every situation and at the same time to wait for the next scene in the story, and we can present mathematics and geometry courses, for example, in every semester based on one story context. Furthermore, such a braiding approach could enable students to improve their beliefs about the importance of geometry in the real world.

The main goal of Civil's project was bringing everyday mathematics, mathematicians' mathematics, and school mathematics together in the classroom. The researcher mentioned the main settings and features of the suggested approach, which was aimed at enabling students to analyze information within a certain context, explain their thinking, listen to one another considering different points of view, collaborate in small groups and take responsibility to justify their conclusions. One of Civil's reflections in analyzing the students' participations is very important and related to the present study of attitudes towards geometry; she stated that students' beliefs and values play an important role in their behavior when faced with a mathematical task.

This emphasizes the multicomponent definition of attitude as students' emotions, beliefs and behaviors, which show the possibility of changing students' behavior especially in their reluctance to participate in geometry class through considering their negative emotions and false conceptions towards geometry and geometric proof.

In another attempt to investigate how connecting mathematics problem solving to the real world, De Corte, E., et al. (2000) presented a mathematical modeling approach that forms a bridge between mathematics as abstract structures and as a tool for describing aspects of the real world. In their paper, they stated the power of the suggested model to modify the belief and feeling of many students that mathematics is irrelevant in relation to their everyday experiences. The following figure represents this mathematical modeling approach:

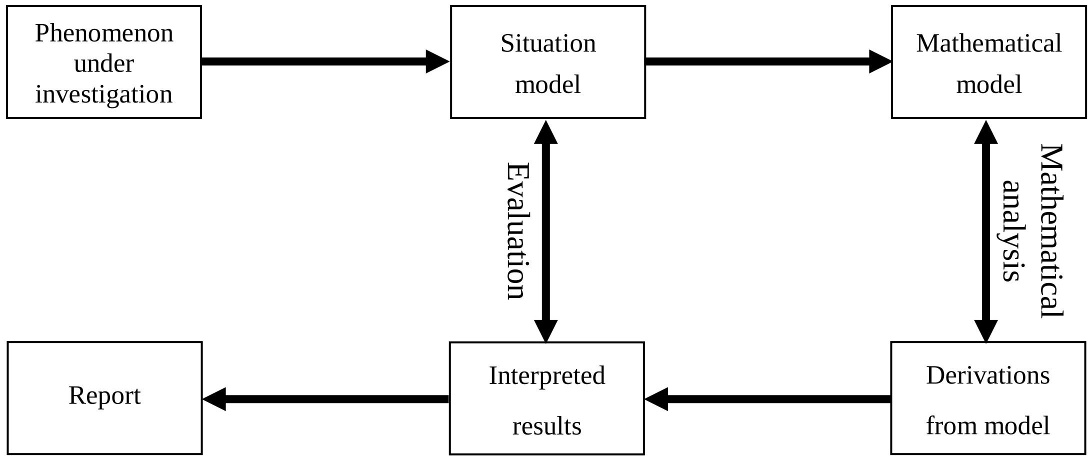


Figure 1: Schematic structure of modeling process.

(De Corte, E., Verschaffel, L., & Greer, B., 2000, P. 71)

Casciato (2003) reported the project of the University of Chicago School Mathematics (UCSMP) which aimed to improve the mathematics curriculum and instruction for all school students in the U.S. and in particular to enable students in grades K-5 to explore geometry through the everyday mathematics activities. He stated the plain effect of the project as being that everyday mathematics does not require students to memorize mathematical concepts or operations. Thus, students have the tendency to remember facts easily when they are active participants in the learning process in the classroom.

Furthermore, learning through exploration not only challenges students to think on their own, but it keeps them interested in the learning tasks. Beyond that, he argued that if parents and teachers collaborated together to make the Everyday Mathematics Project work, everyone would benefit. The great change that could be achieved from this method is that students will not be learning just to pass the final examination but they will gain a deep understanding how mathematics could fit into their everyday life.

Linchevski and Williams (1999) reported an instructional method with the aim to address the cognitive gaps in 6th-grade students' mathematical development who had not yet received any instruction in negative numbers, using intuition from everyday life in making a transfer from their number concept to include the new mathematical concept of negative numbers through 5 one-hour-long sessions. The researchers in this report were directed by the concept of realistic mathematics, which was presented by Treffers (1987). The method aimed to utilize students' everyday sense and intuition to connect the newly acquired knowledge to what students already have, which reflects their concerns to activate the principles of cognitive theories of learning. Linchevski and Williams articulated that in "*situated learning*" and in a social context, the transfer of knowledge is considered to be a challenge, and stated that when they bring realism into the classroom, they cannot closely reconstruct the same social situation in which the students experienced the reality outside school. Thus, they were looking for situations congruent to those in real life, in order to design activities which engage the students in repetitions of familiar experiences from outside school which seem to be relevant to the mathematics curriculum. They finally summarize the role of their model into three tasks:

a. Focusing on the intuitive gaps in students' mathematical development which we think call for the use of extra-mathematical knowledge to support reification.
b. Questioning the authenticity of realistic contexts put forward in the classroom situation.
c. Identifying the need for shifts in meaning in the classroom tasks put forward, from intuitive meanings to mathematical meanings.

(Linchevski, L. and Williams, D., 1999, P. 133)

The term '*situated learning*' has been found again through reviewing the literature in an article by Handley et al. (2007), which stated that situated learning theory considers learning and acquiring knowledge as processes which are interrelated and integral to everyday situations in the workplace, family, and other social contexts. It concerns not only the cognitive domain as in cognitivist learning, but also indicates that learning cannot be isolated and then studied as if it were a discrete activity. The situated learning gets its importance from the fact that those activities may relate to the use of language, making definitions, and behavioral responses, in addition that learning is a normal process of everyday practice. The researchers argued in this article that operationalizing situated learning is a difficult and undeveloped area in the literature.

In a study that sought to engage secondary students in real-world tasks that challenge them to investigate these tasks, Stillman (2006) followed what is known as the "*investigative cycle approach*", which was proposed by Kader and Perry (1994). It begins with posing a problem, collecting and analyzing data then interpreting and sharing results. Data in this study was collected from students' task sheets, audio-taped interviews, observations and reflections. Stillman stated that what should be considered in designing such tasks are the negative experiences and attitudes that secondary school students have in mathematics when they were trying to do mathematics and got the first line wrong.

Also, one of the standards that Stillman took into account is "*Task Scaffolding*" which is defined as:

> *"The degree of cognitive processing support provided by the task setter enabling students to solve complex tasks beyond their capabilities if they depended on their cognitive resources alone"*
>
> (Stillman, 2006)

Stillman takes into account both technology assistance and technology type as the factors that contribute to task scaffolding besides task structure, and stated that the use of technology like calculators in his study might stimulate higher-order thinking in investigating real-world situations.

That reflects the view that using an appropriate instructional technology might not only help in modeling real-world mathematical situations but could also activate the role of learning theories, because the term scaffolding refers to one of the constructivism principles, which shows one of the reasons for choosing the use of technology in the present study context.

Another study conducted by Pierce and Stacey (2006) aimed at enhancing the students' views about mathematics by engaging them in a real-world context. The study revised one of the oldest terms in educational psychology, the "*halo effect*" which goes back to Thorndike in the 20th century. It showed how the use of real-world mathematical problems takes advantage of the 'halo effect' to improve students' attitude towards learning mathematics. The results of the analysis of interviews, reports and a survey from secondary school teachers indicated that most teachers commented on various aspects of pleasure that students have, which are associated with mathematical tasks, namely the power of students' ability to use mathematics that has relevance to their lives, engaging them in an enjoyable learning environment, and solving problems related to their interests. Seven teachers from five secondary schools participated in the study. Pierce and Stacey reported four tasks focused on the algebra section of the curriculum, for instance, in one of these tasks the teacher started by inviting students to meet at the local McDonald's for breakfast. During this meeting the teacher had them take a photo with a digital camera of the McDonald's sign. Before beginning the task, they downloaded the photos to computers and edited them. These photos of the McDonald's arches provided the students with motivation to study curves.

Furthermore, the students commented positively about using a real-world context for learning mathematics in different ways; for example, one student commented that mathematics becomes more meaningful, another comment seems to be purely emotional when a student referred to the mathematics class as being pleasant and having brought back memories of childhood. The researcher reported the most important stage in designing such tasks, which is the formulation stage of mathematical modeling, in which the teacher/designer should consider what mathematics will be used during the task in addition to the planned careful scaffolding.

The previous review shows the importance of operationalizing the principles of constructivism theory, in order to develop learning environments that transfer real-world mathematical situations into the classroom. In the following section, the principles of both cognitive and social constructivism, in addition to one of constructivist learning models utilized in the present study will be presented.

2.4.1 Constructivism Theory

The constructivism theory depends on ideas of Dewey, Piaget and Vygotsky. Also, constructivism is a concept that depends on two major historical standpoints that date back to Jean Piaget (1896-1980) and Lev Vygotsky (1896-1934), Piaget considered mental development as a process that proceeds through assimilation and accommodation. While Piaget was concerned with the cognitive development, Vygotsky's ideas emphasized the importance of knowing processes related to community and culture, in which knowledge is shaped through collaborative actions. Both Piagetian "Cognitive" and Vygotskyan "Social" constructivism stated the main theory's features. Therefore, knowledge in constructivism is defined as the product of our own cognitive acts and experiences.

Also, in constructivist learning, students construct knowledge through interactions with the content, the learning environment, class partners and with the teacher. On the other hand, personal interpretations play an important part in these processes (Kanselaar, G., 2002; Confrey, J., 1990).

In socio-constructivism terms, Ross (1998, p. 253) argued that every student can understand and visualize mathematical concepts in their own ways, but these concepts, definitions and algorithms require a commonly accepted language that serves as a means of communicating thoughts and conclusions. Leading from that, the researcher in the present study wants to refer to one of students' common difficulties in geometry class; despite understanding the geometric theorems and proofs, it is difficult to write down their understanding into words or to express it in the correct mathematical language. The students conclude these negative experiences in one common comment addressing their attitudes: "I am not a math person!"

In this regard, it is obvious that socio-constructivism is relevant in overcoming these difficulties and attitudes in geometry and geometric-proof learning. Also, any language, as well as mathematical terms, requires a social context in order for it to be used and to evolve.

2.4.1.1 Scaffolding and Zone of Proximal Development

The two most featured terms of social constructivism stem from Vygotsky's ideas. According to Vygotsky, the Zone of Proximal Development "ZPD" is defined as when learning awakens a variety of internal developmental processes that are able to operate only when the learner is interacting with others in his environment or in cooperation with his peers (Vygotsky, L., 1978, p. 90). Wood and Wood (1996) clarified that ZPD refers to the gap between what students can achieve without help, and what are considered as learning difficulties but what they can achieve under guidance from the teacher or in collaboration with more capable peers.

The authors linked the concept of ZPD and another of Vygotsky's constructivism terms, that of so-called "Scaffolding", which can be seen as the attempt to support the learners to close the gap mentioned before through teacher's guidance or effective collaboration (e.g., see figure 2) in different ways such as engaging students in activities that form a bridge between the existing knowledge and the targeted goals in the new learning situations, providing appropriate instruction and help, enabling students to play active roles in contributing solutions to problems and transfer the responsibility of students from just achieving knowledge to helping other partners to achieve learning goals (Wood, D., & Wood, H., 1996; Bliss, J., et al., 1996).

Anghileri (2006) stated that "scaffolding" acquired its name because it is ultimately removed once the learner can stand alone in a learning situation, but is re-erected if the learner needs further support in another phase.

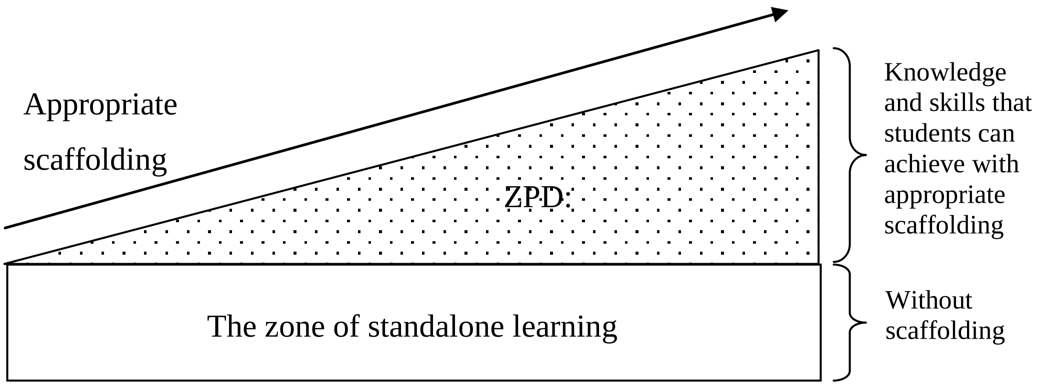


Figure 2: The relationship between the concepts ZPD and scaffolding

Anghileri (2006) clarified two kinds of scaffolding: step-by-step or so-called foothold scaffolding where questioning plays the most effective way to proceed and slots scaffolding or so-called ‘hints’ that also depends on questioning but more narrow or close-ended questions with only one answer that fits.

The interactions that provide students with the appropriate step-by-step help and hints could be organized into several strategies which could play a part in the present study context such as:

1. Imitating ideal behaviors of the teacher or more capable peers

2. Offering appropriate feedback

3. Structuring not only cognition but also beliefs

4. Organizing and justifying explanations and beliefs

5. Inviting and encouraging students to take specific action

6. Inviting and encouraging students to give specific linguistic responses

(Anghileri, J., p. 34)

The following points summarize the related principles of both social and cognitive constructivism in the present study context:

1. Knowledge should be actively constructed by students

2. Learning is both an individual and social process

3. Learning is a process in which sense-making of the real world takes a part

4. Language plays an important part in learning and sharing thoughts

5. Learning should occur and be enhanced with situated meaningful activities

6. Students' interpretations address the process of constructing knowledge and beliefs

7. Students should be motivated and purposively participate in order to achieve new knowledge

8. Learning situations should reorganize the cognitive structure to assimilate the new knowledge with previous knowledge (Simon, S., 2004; Refaat, 2005).

According to the principles of both cognitive and social constructivism, many studies conducted various learning models that vary in the phases and procedures of learning (e.g., Nuhoğlu, H. and Yalçin, N. 2006; Refaat, E., 2005; Liew, C., and Treagust, F., 1998; Palmer, D., 1995). These models were dependent mostly on the ideas of Piaget and Vygotsky, and aimed to enhance students' achievements and understanding in different learning topics such as physics, chemistry and mathematics. The following constructivist learning model is an example that could be related to the context of the present study.

2.4.1.2 The Predict-Observe-Explain (POE) Model

This model presented by Gunston was aimed at stimulating students to think about the presented concepts, enabling them to participate in a three-stage learning situation starting with predicting results and discussing a specific topic, getting them involved in experimentation and to collect data supported by the teacher, and writing down their observations. Thus, in the last phase students attempted to describe their observations and explain the agreements and/or discrepancies among their predictions and observations (White, R., & Gunstone, R., 1992). The present researcher wants to highlight the importance of the last phase mentioned above. Students should be invited to think about the reasons for any discrepancies in their conclusions, and then receive interpretation feedback from their teacher, which is not just a comment on the students' participation in the form of 'well done' or 'wrong try again', but actually clarifies that the interpretation of a wrong answer might lead to the right one, because the simple kind of feedback might be one of the sources of frustration in the classroom when the student finds that his participation has varied from other classmates' answers. This might constrain the student in participating again and might lead them to express a negative attitude like "I am not a math person!"

Considering the importance of sharing mathematical ideas and according to Johnson and Johnson (1990), cooperative learning is one of the social contexts, which could be a practical way of proceeding constructivism ideas that emphasize the importance of enabling students to communicate mathematically. The following section presents further details about cooperative learning, its importance, its elements and one of cooperative learning structures which plays a part in the present study.

2.4.2 Cooperative Learning

The review of literature revealed that cooperative learning is one of the most successful teaching strategies that present the benefits of the ideas of constructivism theory. A considerable amount of literature referred to the possibility to use various learning activities and to engage students of different levels and different abilities in a cooperative learning context (Kagan, S., 1994; Kagan, S., 2001).

However, many definitions of cooperative learning referred to engaging students in small groups, working together to increase their academic achievement and to enhance the value of interactions among them (e.g., Nichols, J., 1996).

Zakaria, E., et al. (2010) saw cooperative learning as more than just as an effective strategy for students' achievement, in which students work pairs or in small groups, with different levels of ability, and use a variety of learning activities. This could improve not only their understanding of mathematics but also improve their attitudes toward it.

2.4.2.1 Importance of cooperative learning

Zakaria, E., et al. (2010) explained that cooperative learning might be one of the methods that students need to overcome the well-known phenomenon of frustration among them and even their teachers in mathematics. The wide-ranging use of cooperative learning showed its effectiveness due to numerous factors. Slavin (1992) and Zakaria (2010) determined some of the reasons explaining the importance of cooperative learning such as enhancing and deepening students' learning of formal curriculum, performance, and long-term memory, while other affective effects include positive attitudes, self-confidence in learning new content and improving social skills to work productively with others. Furthermore, McCloskey (2002) argued that when teachers use students as co-teachers in cooperative learning, students can use thinking skills as they compare different views in order to come to agreement and prepare information to present to the rest of the class. The results of Nichols (1996) showed a significant improvement in students' achievement and motivation in geometry class, so in comparison to students in traditional instruction group, the students in cooperative learning conditions showed higher levels of geometry achievement, more goal-oriented learning, more positive beliefs about their abilities in geometry and deeper cognitive processing strategies.

The researcher here wants to emphasize the relevancy of the real-life geometry situation in the present study with the cooperative learning context, due to Dewey's views of the importance of integrating the values and skills of the "real-world" in school lessons and activities (Fryer, W., 2003).

In addition, Dewey argued that discussion-based teaching and the beliefs about the importance of engaging students in direct experiences and simulations might form an appropriate context for problem-centered instruction (Smith, B., and MacGregor, J., 1992), which might stimulate students and preserve their motivation to work together and share their ideas, in order to find optimal solutions.

2.4.2.2 Elements of cooperative learning

Related studies and literature on cooperative learning refer to some main elements of cooperative learning such as:

1. Social skills that make cooperative learning more complex than individualistic learning as students must engage simultaneously in tasks and teamwork,
2. Group processing is considered as one of the basic elements of cooperative learning.
3. Successful group work is affected by whether the group reflects on how well they are performing or not.
4. Cooperative groups are alive with two-way interaction. Students should do real work together in which they promote each other's success by sharing resources and helping, supporting and encouraging each other's efforts to achieve.
5. Face-to-face interaction is successfully structured when students enhance each other's learning.
6. The feeling among a group of students that they need one another to achieve. When one student achieves the others benefit. In other words, students perceive that they are linked to each other in a way that one cannot succeed unless everyone succeeds.
7. The feeling among a group of students that each member is responsible for his own learning as well as that of the group mates.

(Johnson, W. and Johnson, T., 1990; Zakaria, E. and Iksan, Z., 2007)

2.4.2.3 Think pair and share

Think pair and share is one of the cooperative learning structures also considered as one of the social constructivism applications. This is a three-step cooperative strategy. In the first step, the teacher poses a question; preferable questions are those that require analysis, evaluation or synthesis and give students the time to think through an appropriate response. After this waiting period, students pair up and share their conclusions, in addition to allowing feedback on their participations. Through the last step, student responses are shared in every group or between every pair and then with the entire class during a discussion (Spencer, K., 1999, p. 20). However, engaging students in a social context and real-life situations is not enough; in order to meet the needs for further geometrical understanding, an appropriate environment is needed. As Kemeny (2006) explained, establishing a connection between instruction and real-world requires hands-on activities. Consequently, designing instructional environments that evolve students' experiences from just realizing real-life geometry situations to the level of doing geometry and proving becomes the new challenge. Many studies have been conducted to investigate the facilities and the effectiveness of using several learning environments to overcome the inadequacy in mathematics teaching and learning, especially in geometry (e.g., Noss, 1988; Refaat, 2001). However, studies are scarce that engage students in real-life geometry situations to overcome difficulties in geometry and geometric-proof learning as well as improving their attitudes towards these subjects in mathematics classrooms (e.g., Stillman, 2006; Pierce & Stacey, 2006; Duatepe-Paksu; Ubuz, 2009). That shows the relevancy of developing a geometry learning environment in the context of the present study, which could enable students to participate actively in establishing new geometric knowledge.

Consistent with this view, previous studies indicated the effectiveness of using educational technology in mathematics education. Pea (1987) and Smith (1999) stated that the role of computer use not only amplifies students' ability to learn, but it also has the facilities to change learning methods; it could be used as a cognitive tool as well as play the role of intellectual partner. This might reflect the benefits from both cognitive and social constructivism, e.g., Piagetian and Vygotskyian constructivism.

Refaat (2005) conducted a study that attempted to develop a multimedia-based program, and investigated the concepts of what is called "*Techno-constructivism*" that integrated the aspects of both educational technology and constructivism theory, in order to overcome students' anxiety arising from geometry learning situations. In addition, the results indicated the positive effects of using the developed software on students' achievement of the included geometric concepts and theorems.

Mehlinger (1998, p. 12) discussed further aspects of using such educational technology, in that it encourages cooperative learning and increases the interaction between the teacher and students, in addition to its positive effects on students' attitudes. Numerous studies investigated the most effective and related technology and its roles in mathematics education, Dynamic Geometry Software in problem posing, understanding and solving, and particularly in geometric problems and situations.

2.5 What is Dynamic Geometry Software 'DGS'?

Kortenkamp (1999) gave an answer to this question by stating that it shaped new opportunities for computer use in the field of mathematics teaching and learning and especially in regard to geometry. While it enables the user to construct accurate geometric elements such as lines, circles, intersections, loci and traces of these objects that could be displayed on the computer screen, the fundamental advantage of dynamic geometry software, in comparison to the traditional ruler and compass, is that it magnifies the ability to explore (e.g., rediscover) the behavior of a construction by dragging the included elements in every geometric configuration.

In his book '*How to solve it*', George Polya stated two roles of discovery:

> *"The first rule is to have brains and good luck. The second rule is to sit tight and wait till you get a bright idea"*
>
> (Conway, H., 2004, p.172).

Nevertheless, the present researcher expects that through using dynamic geometry software, the second rule of discovery in mathematics and particularly in geometry might be to some extent changed.

While students can pursue the same method of discovery as followed by the first geometer, by enabling them to play an active role instead of just inviting them to "*sit tight*", and by engaging them in experiments instead of "*waiting till they get a bright idea*", dynamic geometry software can enable them to get bright ideas, especially those required to start the proof (e.g., the strategy of proving one geometric statement) through dragging, measuring and observing.

Kortenkamp (1999) concluded that the most important aspect of using dynamic geometry software is the fact that you can explore the behavior of the geometric configuration by moving its elements as the user can see what parts of the construction change and which remain the same. Thus, students can obtain a deeper insight and understanding about the geometrical construction and its properties.

Jones (2001) emphasized the same aspect; with the possibility of dragging the included elements, the students get the impression that the geometrical configuration is continuously deformed, and while animating geometric elements, they observe that different parts of the configuration respond by preserving the geometrical relationships, else they might observe specific behavior of the configuration while reaching one element to a specific position.

Although benefiting from these facilities might stimulate students' motivations to start the journey of learning with the guessing reaction that mentioned by Vygotsky (so-called Aha-reaction), Vygotsky indicated this "aha" reaction does not mean that they have discovered or deduce the new knowledge or new relations in that form of a sudden insight (Vygotsky, L., 1978, p. 45).

However, such a simple psychological operation could be operationalizing as the first step on the way of making conjectures and then justifying conclusions. In addition, through using dynamic geometry applets, the students can test whether their constructions work in general or whether they have discovered a special case (Giamatti, C., 1995, p. 450).

Jones (2001) concluded that the dynamic geometry learning environments appear to have the possibility to:

1. Engage students in a 'direct experience' that enables them to learn geometrical concepts and theorems and closes the gap between geometrical construction and deduction.
2. Enable students to making distinction between what varies and what is invariant in a geometric configuration.

3. Enable students to get the idea of proof using pre-scripted dynamic geometry applets. For example, with dynamic geometry software, they are likely to control the produced applets in order to provide the following situations in experimenting and investigating geometric configurations:

    a. Points that cannot be dragged could be completely hidden or displayed in a draft mode, for instance, points of intersection.

    b. Dragging effects and conditions that are determined by the applets developer, for example, the decision about what happens to an element such as a point placed arbitrarily on line segment when one end of the line segment is dragged.

2.5.1 Why Dynamic Geometry in the present study?

This section reviews some of the related studies that attempted to harness the benefits from the use of dynamic geometry software, in order to improve educational variables relevant to geometry learning. Also, the reviewed studies indicated various impacts of using dynamic geometry software. For instance, its effects on problem posing and solving (Christou et al., 2005), understanding geometric concepts and theorems (Almeqdadi, 2000), discovering and conjecturing (Aarnes, and Knudtzon, 2003; Furinghetti and Paola 2008; Habre, 2009), improving achievement and attitude towards mathematics (Phonguttha et al., 2009) and its impacts on developing reasoning and proof abilities (Jiang, Z., 2002). Güven and Kosa (2008) concluded that the dynamic nature of DGS is not only efficient at enabling students to learn geometric concepts and to explore geometric relationships easily, but it also has the power to develop their spatial skills.

Furthermore, Almeqdadi (2000) clarified the facility of dynamic geometry software to magnify geometric configurations so that it becomes easier for students to recognize geometrical proprieties. Almeqdadi referred to another possibility that might relate geometry-learning tasks to the real world of students: using dynamic geometry activities can engage students in an effective learning environment that provides them with a good simulation of geometric figures and phenomena that they might face outside the classroom, which is much closed to the real-life situations. Jones (2001) summarized the didactical side of DGS's facilities which could help students to participate in an interactive learning environment that might reduce the gap between geometrical construction and deduction, and gain a more meaningful idea of proof and proving. In constructivism terms, Hay and Barab (2009) assumed that "… by the degree of active learner engagement as well as the assumption that learners have the ability to create meaning, understanding, and knowledge. Students are not passive receptacles of the knowledge that teacher impart".

In the following there is a review of the related studies which played a vital part in all procedures of the present study and in developing its treatments. This review will be followed by the concluded rules of dynamic geometry software in introducing students to real-life geometry, so as to improve their attitudes towards geometry and geometric proof.

Lee and Chen (2008) presented a problem-solving activity using computers as an attempt to find the appropriate channel to transfer students and evolve them from the level of making conjectures to the level of writing proof as a means of explanation for their own findings; it also seems that the authors had a hidden aim in the article concerned with students' beliefs about the functions of proof, as they mentioned that students in traditional teaching cannot feel the necessity of writing proofs, and they prove just because the teacher asked them to do so. The activity presented in the article was divided into three phases: introduction, exploration in groups and reporting of conclusions. The exploration phase aimed at leading students to formulate conjectures and reporting the solutions in the next phase.

Christou et al. (2006) conducted a study that used DGS as a tool of mediating students' strategies in solving, posing and extending problems. The study investigated how DGS enables 6 pre-service teachers to benefit from it in the process of modeling the geometric situations, experimenting, conjecturing, and generalizing. And it aimed also at benefiting from the role of this instructional technology in making surprise and cognitive conflicts through dragging and measuring facilities that enables students to use their visualization and spatial reasoning abilities. Only two geometric problems were presented to the students; the first problem is a real-life geometric problem as well known as the airport problem, while the second is more abstract and investigates the figure formed by the bisectors of the interior angles of a parallelogram. Christou concluded that the study addressed ways in which dynamic geometry could be integrated with new geometric problems in geometry class; those which do not usually appear in the traditional geometry textbooks. The results also showed that the presented problems established accurate views of the geometric situations and helped students to actively participate in exploring geometric configuration and the included relationships, gain a deeper understanding and visually confirm or reject their conclusions.

In Germany, Gawlick (2002) conducted a study to examine the impact of dynamic geometry software on geometry learning in comparison to the paper-and-pencil environment in regular classrooms. 214 seventh-grade students from three senior high schools participated in the study experiments and were divided into three groups: dynamic geometry software, paper-and-pencil and control group, where the treatment topics included concepts such as perpendicular and angular bisector, the circumcenter and the circumcircle, the incenter and the incircle of a triangle. All participants underwent a pre- and post-test. The results indicated that dynamic geometry software was helpful to generate conjectures, but did not find arguments supporting them. So that a significant difference between dynamic geometry group and control group was found as well as significant differences in achievement and strategies between paper-and-pencil and dynamic geometry group. The researcher recommended, *"... teachers must be put into a position to develop new teaching sequences.*"

Groman (1996) conducted a study that used three examples to examine how Geometer's Sketchpad is used in making and testing conjectures in secondary school mathematics. The results showed that students had positive attitudes towards constructing the geometric configurations in their own, in addition to the positive participation and reactions from students and even the teachers themselves. That is consistent with the results of another study conducted by Yousef (1997), which showed that one of the well-known dynamic geometry software 'Geometer's Sketchpad' evolved students' attitudes towards geometry in comparison to the traditional geometric tools that is used in the control group that studied with the traditional instruction.

Furinghetti (2008) explained that the possibilities to motivate students to prove through dynamic geometry activities are different from those in the traditional methods, while the motivations provided in such interactive activities are similar to those that some mathematicians have. Accordingly, when students play an active role in rediscovering geometric concepts and theorems by themselves, they will not only be able to realize the verification function of proof, but also be motivated to explain why their own conclusions are true, and consequently feel the importance of geometric proof. Kemeny (2003) agreed with that and clarified that in a learning environment that depends on dynamic geometry software, proof has a different meaning as it satisfies students' motivations to explain why their own conjectures and conclusions are still true beyond the variation and the examples presented during dragging on the screen. Without such an inductive process, students might see proving as a meaningless task with the sole purpose to reach a truth that they are already convinced about.

Govender and de Villiers (2002) conducted a study that examined the effects of Geometer's Sketchpad on 18 future mathematics teachers' understanding of the nature of geometric definitions, in addition to increasing their ability to formulate and evaluate definition. The study also concentrated on changing students' beliefs that there is only one correct definition for each mathematical or geometrical object. Also, the results showed that most of the student teachers were able to recognize the correct definitions and can describe the uneconomical definitions, but half of them were able to change them to correct and economical definitions. Furthermore, the qualitative analysis of students' participation showed that the possibilities of dragging, measuring and the discussions through activities enable them to improve their definitions and the mathematical language required to formulate them. De Villiers (2003) shed light on the idea of discussing and sharing geometric thoughts and conclusions. He also clarified the effectiveness of DGS, stating that

> *"It enables students to experiment through unlimited variation construction and makes conjectures. So that the next task is to investigate whys their conjectures are true, while the dynamic geometric construction is simply sufficient for convincing with the trueness of conjectures".*

In Japan, Nunokawa and Fukuzawa (2002) conducted a study aimed at analyzing students' understanding of geometry problem situations and their "why" questions during their problem-solving processes in order to provide insights into the nature of such questions. The participants had no experience with dynamic geometry software. The data of the study gathered during research by the second researcher Fukuzawa, were aimed at exploring the roles of dynamic geometry software "Cabri" in developing ideas to prove geometry problems. Two pairs of Japanese ninth grade students participated separately in five sessions and tackled one geometry problem using "Cabri" in each session followed by an interview. The results obviously revealed that the dynamic geometry software assisted students' exploration and understanding of the problem situation.

In a study of the effect of using Geometer's Sketchpad "GSP" on students' performance and mathematical thinking along with initial exploration, Abu-Bakar, et al. (2009) used a set of lessons on the topic of quadratic functions in a national secondary school in Malaysia. The lessons module was distributed to the 45 students in the experimental group to be used as a guide through the learning process in the class. In the first phase, the students were encouraged to explore and become familiar with the use of the GSP and its facilities. In the second phase, the students were introduced to the concept of the quadratic function and problem-solving sessions related to the topic. In the last phase, an achievement test was administered in which students solved given problems but without using the GSP. The results of the experimental group in comparison with the results of 47 students in the control group showed that there were no significant differences in performance between the control group and GSP group. The students' attitudes towards the suggested approach were measured according to four dimensions (e.g., four levels): enthusiasm, enjoyment, anxiety and avoidance. Unfortunately, the authors reported that there were no overall differences in students' attitudes between the control group and GSP group. The avoidance dimension refers to GSP group students' perceptions that the suggested approach was a waste of time and an unproductive effort. The authors supposed that the time constraint might be one of the reasons for these results.

In the U.S.A., another study was conducted by Hull and Brovey (2004) to describe the effects of using dynamic geometry software on students' achievements and attitudes towards geometry. Sixty-eight ninth-grade geometry students were taught using the GSP in 50-minute classes twice a week while studying the "Circles" unit over a period of three weeks. All students responded to a questionnaire, which was administered at the start and end of study experiments in order to survey students' attitudes to geometry in addition to a paper-and-pencil geometry unit test. The results indicated no significant differences in students' achievement in comparison with the results of other studies. Even students' attitudes were indicated as showing no significant change according to data gathered from the questionnaire.

De Villiers (2000) conducted worksheet-based interviews with 14-year-old students aimed at determining their levels of conviction and needs for explanation within the learning context using dynamic geometry activities. The interview engaged students in an example that investigated the sum of the distances from a point to the sides of an equilateral triangle, in the cases when the point lies inside the triangle and outside it. The presented extract in the paper shows that the students need to become more convinced whenever they have the possibility to engage in further experiment. The researcher stated that students' levels of conviction seemed to be much higher than expected; in addition students might not have reached these levels of conviction so quickly and so easily if they just had paper and pencil. De Villiers also indicated students' desire to attempt their own explanations. Furthermore, they were able with guidance not only to make a conjecture, but also to explain why the conjecture was true. However, he reported the difficulty of concluding that all students had considered the proof as a meaningful activity.

Aarnes and Knudtzon (2003) emphasized the processes of reasoning rather than proof writing, and stated that if the formal proof is seen as the most important of these processes, that could simply lead to most students losing their interest in and curiosity for geometric proof. The researchers in this paper presented the same example presented by de Villiers (2000) for secondary students, using dynamic geometry software in discovering relationships, making conjectures, discussing and generalizing their conclusions. The paper showed in an indirect way the importance of questioning during experimenting and conjecturing; thus, questions were posed such as: "*What happens if.., and why this is so?*" To some extent, this conclusion is consistent with the results of the study conducted by Habre (2009) that emphasized the role of the teacher in guiding students during their experimentation with dynamic geometry software. As the continuous variations of geometric configurations may sometimes lead to students' confusion, they might then need appropriate support from the teacher in order to develop their thinking strategies. Also, the study was aimed at examining the effectiveness of dynamic geometry software in enabling students to gain a better understanding of geometrical concepts and theorems, in addition to making and determining the trueness of conjectures.

Christou et al. (2004) conducted a study that investigated how three prospective primary school teachers explored problems in geometry and how dynamic geometry software could close the gap between their constructions, experimentations, and conjectures on one hand and proof on the other. When students start writing proof, they stop working on the computer. Thus, the main aim of conducting the study was to utilize dynamic geometry effectively to introduce proof in a meaningful way that would enable students to recognize functions of proof other than verifying the correctness of one theorem. The suggested approach consisted of three phases: the first phase is before proof and engages students in an exploration of constructions with the help of the facilities of dragging and measurement, which led them to formulate their own conjectures about the possible solutions. Proof was the second phase, in which the students were encouraged to explain the reasons for the correctness of their conjectures and possible solutions, where the aim of this phase is to enable students to pass from exploratory geometry to deductive geometry, which could bridge the gap mentioned earlier.

The last phase was concerned with extending proof to similar problems in two different categories; problems that have similar context, and problems that required the same reasoning processes. The importance of the last phase was to insure that the transfer of learning has done, and that students can use the proving process in new problems with different structures.

Furinghetti and Paola (2008) conducted a case study with a pair of two students working together. The main goal of the study was consistent to some extent with Christou et al.'s (2004) study mentioned previously, which also aimed to close the gap between the level of experimenting, conjecturing and the level of writing the proof. Although, Furinghetti and Paola (2008) argued that the approach they presented provides students with motivations similar to those mathematicians have, and is more effective than the form "prove that …", they also clarified that working in groups and with an open problem strategy transferred the responsibility from the teacher to students, fostered creativity and led them to produce divergent thinking. The researchers reported the importance of the phase after making conjectures, whereas the students' focus changed from observing the screen and writing conclusions to the communicating and sharing ideas in order to justify and prove.

Here, the present researcher wants to add emphasis on the importance of the social context mentioned earlier in the part that discussed constructivism theory in order to evolve mathematical language the students need in expressing their ideas and writing the proof.

Also, Furinghetti and Paola (2008, p. 403) summarized the phases of students' reasoning and referred to the "Aha-reaction" mentioned before by Vygotsky; these phases started with reading and translating the problem into a geometric configuration using 'Cabri' dynamic geometry software in an accurate interpretation for statements. The second phase was concerned with introducing students to a kind of dragging called 'wandering dragging' in order to produce conjectures which are close to those experimental sciences that use empirical methods; the Aha-reaction was mentioned in this phase as the moment of discovering properties. The next phase also dealt with dragging but sought a way of justifying 'proving' the conjecture; the way of thinking here is closed to the analytic method. The authors ended with a final comment that students will go back to using paper and pencil instead of 'Cabri', which could be considered a new gap between using dynamic geometry in reasoning processes and should be left out of the phase of writing the proof.

Olivero and Robutti (2007) conducted a study aimed at investigating how measuring tools affect the shaping of the proving process and how students' ways of using the measurement tools in the proving process could be classified. This study presented 15-16-year-old students from schools in Italy and United Kingdom with open problems in an attempt to integrate geometry teaching and using dynamic geometry software tools. The students who participated in this study were asked to work in pairs with Cabri and trying to formulate conjectures and to prove them. The data were collected using videotapes, observations, and field notes of students' pairs. Three examples in the study experiments were discussed. The first two examples reflected "the origin of conflict that may appear when measurements are read as exact and the Cabri figures are not re-interpreted from theoretical point of view" while the last example presented "the productive interaction between the spatio-graphical and the theoretical field". The results showed that measuring in a dynamic geometry environment is classified according to three roles: the first was measuring and conjecturing, the second was measuring and proving, and the last was an integration of different modalities of measuring.

In a study for the effect of dynamic geometry software on student mathematics teachers' spatial visualization skills, Güven and Kosa (2008) developed some activities using Cabri 3D software, which were implemented for 40 student mathematics teachers in Karadeniz Technical University in Turkey, for at least 1.5 hours in a week over eight weeks. The study was conducted using the one-group pre-intervention-post experimental design. The results of the study show a significant difference in participants' spatial skills due to the use of dynamic geometry software 'Cabri 3D'. The researchers reported an unexpected and interesting point that the students' spatial skills were low. This might be attributed to many factors, such as the traditional methods in Turkey which present three-dimensional geometrical objects on the blackboard in a two-dimensional format. They explained that such a limitation does not enable students to create and manipulate three-dimensional models, and accordingly they do not have the appropriate context to develop their spatial skills. This study might seem to be unrelated to the present study context as it used Cabri 3D software.

However it sheds light on the importance of the spatial skills and creating visual models for the geometrical concepts and relationships, which could facilitate the process of building the cognitive structure in students' minds, and beyond that, it could enable them to reach the level of visual deduction. On the other hand, this study also showed one of the student teachers' characteristics of low spatial skills, which might exist in many of the faculties of education around the world, where the students who will be mathematics teachers in the future and supposed to have a high level of spatial and mathematical skills, and also positive attitudes towards mathematics and geometry. This invites researchers in the field to investigate the conditions of attending faculties of education, and to develop innovative teaching and preparation methods for the prospective mathematics teachers, in order to guarantee 'to some extent' the mathematics learning outcomes in the future classrooms.

The study of Patsiomitou and Emvalotis (2010) investigated the effects of using Geometer's Sketchpad on students' movements through van Hiele levels. Three students, as an experimental team in a group of 14 students aged 15-16 in Greece, were engaged in one- or two-hour sessions of meaningful activities that includes relevant geometrical concepts; the students had no experience with dynamic geometry software, which was used to enable students to make an exploration of and to structure logical mathematical meanings through guided reinvention process. Results indicated that students are seemingly able to transfer the skills and the different ways of looking at the reformulated problem acquired when working in dynamic geometry activities. The students developed their visual and verbal skills. The results also showed the development in students' interpretation and in using the proper terminology to describe the relationships between construction's elements. The researchers finally concluded with their observation of a link between visual ability and formal skills which is essential for the transition from the lower van Hiele levels to the higher ones.

Sinclair (2003) investigated the effectiveness of working with pre-constructed web-based dynamic geometry sketches on offering accurate visual evidence for secondary school students. The study was focused on congruence and parallelism in grade 10 and on similarity in grade 11 in three 45-minute sessions with each grade. None of the participants had worked with dynamic geometry software before. The results indicated that using dynamic geometry sketches alongside good stimulating questions, through experimenting tasks, enabled the students to rediscover geometric concepts and relationships without confusion. This helped them to explore depending on the unlimited variation, and to develop their visual interpretation skills. However, the results also showed that the usability of dynamic geometry sketches was affected by the students' previous learning experiences. The study recommended that teachers should help students find ways to be aware of the benefits of visual reality.

2.5.2 The Combination between real-life geometry and DGS

To conclude, the standards set out by the National Council of Teachers of Mathematics in the U.S.A. emphasized the use of concrete models using dynamic geometry software to explore geometric ideas and realize their usability in real-world contexts (NCTM, 2009). Leading from that, this study included the use of dynamic geometry software in presenting geometric concepts and theorems in real-life situations.

This combination may provide students with an appropriate geometric-learning environment in which they would actively participate in formulating geometric statements and conjectures by themselves, recognizing the idea of the proof. Moreover, the facilities of dynamic geometry software are able to model real-life geometric situations, which bring the geometric concepts and theorems to life and bring the real-life geometric situations into the classroom.

Almeqdadi (2000) mentioned this aspect, stating that:

> *"Using this program will provide students with a good chance of simulation, which is very close to the real-life situations."*

The present study attempts to benefit from this view, in order to develop the various dimensions of students' attitudes towards geometry and geometric proof, engaging students in related and sequenced real-life geometric situations might add more power to the use of dynamic geometry software.

Such a learning context might enable students to use their previous experiences and knowledge to discover and share geometric ideas, which activate the principles of learning theories such as social and cognitive constructivism. In terms of social constructivism, the present researcher expects that using such an approach might engage students in a social learning context, which is essential for improving the language required to express their thinking and writing down geometric explanations. The NCTM's guiding principles indicated the importance of using language to express ideas precisely and organizing mathematical thinking through communication, and stated that "*When students communicate the results of their thinking with others, they learn to be clear and convincing in their verbal and written explanations*" (NCTM, 2009).

2.5.3 Roles of that combination in the present study

The combination between dynamic geometry software and the series of the real-life geometric situations might be the main outcome and the actual impact of the present study, whereas a new geometry learning approach that could be so-called "*Story-Based Dynamic Geometry*" was suggested. Thus, in the following are the roles of the suggested approach that will take part in the present study context in order to improve students' attitudes and overcoming their difficulties; as it helps students to:

1. Realize the relevancy of geometric concepts and theorems to real-life, which could change their false beliefs towards geometry as being an unimportant topic.

2. Carry out experiments and conclude geometric information, which eliminates their negative emotions that appeared in students' utterances such as "I am not a math person!"

3. Visualize and understand geometric concepts theorems more deeply.

4. Make conjectures and formulate concepts and theorems by themselves.

5. Discuss and share ideas and conclusions in geometry class.

6. Convince class partners for each other in geometry class with the trueness of their own conclusions.

7. Get proof ideas and how to start writing the geometric proof.

8. Develop their mathematical language, which is required to explain, justify and share geometric conclusions.

9. Reach the level of formal proof in a smooth transition and according to van Hiele levels of geometric thinking and Hoffer matrix.

These roles will take part in forming the learning phases of the suggested approach in the context of the present study. These phases will be presented in the next chapter.

3 Study Methods and Procedures

The present study sought to develop several geometry learning activities; through this development many procedures have been followed in order to design these activities as well as the measurement tools to determine whether there are significant differences in students' attitudes towards geometry and geometric proof due to participating in the activities of the suggested approach, which was based on a combination of real-life geometric situations that form a short geometric story and benefit of the facilities of dynamic geometry software.

Thus, this chapter presents the important research phases that follow a review of the related studies and literature. It also clarifies the research methodology employed in the present study and provides a deeper insight into the preparatory processes of the treatment material and the measurement tools. Also, this chapter deals with five main parts: the first presents the vision of the suggested approach, its aims, its content, the prerequisite and objectives list of its activities and the learning phases in every activity; the second gives details on how the activities CD-ROM was developed; the third entitles "Instrumentation" sheds light on the steps of designing five measurement tools that include three questionnaires: the first deals with attitudes towards geometry and geometric proof, the second towards learning using computer and the third is concerned with students' attitudes to the suggested approach. In addition there is an entrance test and a geometry achievement test. This part also describes the validating of the measurement tools through a pilot study and presenting them to a group of specialists teaching mathematics in both Egypt and Germany. The judges reviewed the measurement tools for suitability to what it intended to measure, and reviewed the suggested activities for the appropriateness of language and the suitability of the topics in order to improve students' attitudes towards geometry and geometric proof. Part four of this chapter is concerned with sampling. The final part of this chapter presents the main study experiment, ending with the researcher's observation and comments on the study experiments and the required time for studying every activity. The following figure gives an overview of the entire study procedures.

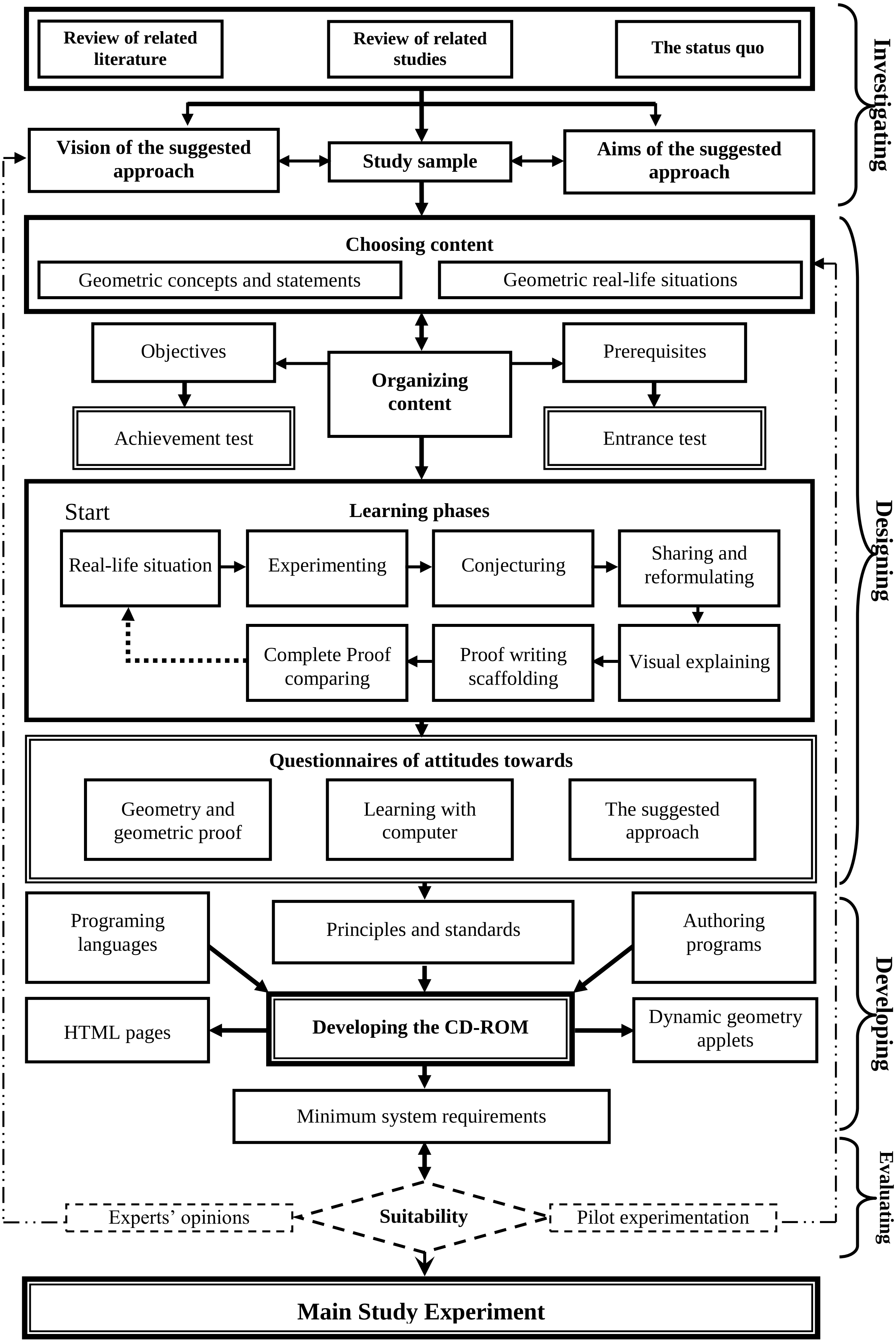


Figure 3: An overview of the study procedures.

3.1 Vision of the Suggested Approach

In order to give a complete vision of the suggested approach and its content, this step is divided into several parts that provide answers to the following questions:

1. What are the aims of the suggested approach?
2. What is the content of the suggested approach?
3. What are the learning prerequisites for studying the content?
4. What are the learning competencies of every learning activity?
5. What are the learning phases in every activity?
6. What are the students’ roles during these phases?
7. What are the teacher’s roles during these phases?

3.1.1 Aims of the suggested approach

The main goal of the present study was divided. Firstly, to develop a suggested approach that starts with a real-life geometry situation and is based on the facilities of dynamic geometry software. Secondly, to overcome students’ difficulties, and to improve their attitudes towards geometry and geometric proof.

This goal can be elaborated in the view of the definition of attitudes (e.g., emotions, beliefs, and behaviors), the advantages of real-life learning contexts and the facilities of dynamic geometry software. This broad goal branched out into the following aims:

1. Enable students to realize the relevancy of geometry to the real world by using a geometry story consisting of related real-life situations, which contains geometric concepts, theorems or problems that need optimal solutions.

2. Enable students to interact with the geometric content throughout experimenting while feeling they can do geometry and formulate conjectures and geometric statements by themselves.

3. Increase students' motivation to investigate why their own conclusions are true, which enables them to feel the importance of geometric proof as being more than just to examine the trueness of a theorem.

4. Reformulate students' verbal and written conclusions through participation in a social learning context that enables them to communicate their thoughts and results.

5. Overcome one of the most difficult steps in geometric proof which is to get the idea of the proof, e.g., how to start proving, based on the facilities of dynamic geometry software and the included programing language scripts.

6. Enable students to go further in explaining the steps of proof in their own words and based on recognizing these steps, visually through sequenced geometric configurations provided in a slideshow.

7. Scaffold students' written justifications and compare them with a complete geometric proof based on the JavaScript picture-hover function.

3.1.2 Principles of Developing the Suggested Approach

According to the related studies and literatures, the following are the principles of developing the suggested approach, based on the facilities of dynamic geometry software in introducing students to real-life geometric tasks:

1. The geometric tasks should be organized according to the principles of constructivism theory with an emphasis on engaging students in learning content relevant to their lives, in order to improve their attitudes towards the importance of geometry.

2. The learning phases within the suggested approach should be designed to be effective in overcoming students' difficulties in understanding geometric definitions and theorems, expressing their thoughts and writing a geometric proof. This might change their attitude towards geometry being a difficult topic in mathematics.

3. The suggested approach should start with a geometric story that includes several situations that require optimal solutions or leads to them making conjectures and formulating geometric theorems.

4. The learning phases within the suggested approach should follow van Hiele's levels of learning geometric concepts and theorems.

5. The suggested approach should provide the students with opportunities to investigate every geometric situation through experimentations using dynamic geometry software.

6. The geometric tasks within the suggested approach should provide the students with opportunities to get the key idea of proof through exploring and observing the variant and the invariant elements in the dynamic geometry configuration.

7. The suggested approach tasks should enable students to realize functions of geometric proof other than just verifying the trueness of theorems.

8. The suggested tasks should enable students to collaborate in their thinking and to develop their mathematical language, which is important for them to convince one another regarding the concluded information.

9. The geometric content of every task should be designed according to the cognitive level of students, especially those who have learning difficulties and negative attitudes towards geometry and geometric proof.

3.1.3 The content of the suggested approach

Seven geometric activities formed the whole story called “Building a New City”, followed by six geometric theorems, which were developed during the period of my Ph.D. scholarship in Germany, funded by the Ministry of Higher Education in Egypt (from 2007 to 2010). Two geometric problems were adapted from De Villiers (2006) and Herrmann (2005) to fit the study context. These problems are “the water tank problem” and the “mirror problem”.

Because the targeted Egyptian students did not have previous experience with dynamic geometry software, it was decided to provide them with ready-made scripted applets that could help them step by step in observing, conjecturing, and formulating theorems and explaining why their conclusions are true.

3.1.4 The geometric story

The following is a summary of the geometric story (Building a New City).

Somewhere in the desert, there is a new triangular-shaped city consisting of three housing areas. The city administration decided to build a water tank to serve a maximum of five housing areas. Beyond the housing areas, no more houses are allowed to be built. The water tank should be at the same distance from all three areas. Hence, the first problem that the administration faced, was how to determine the position of the water tank so that is equidistant from all three housing areas.

Due to overpopulation, the city was needed to be extended by adding a new housing area, which is the same distance from the water tank as the other three areas. What shape do the four housing areas need to form in order for them to be the same distance from the water tank? The Ministry of Culture then decided to build a cinema. Think about a geometrical explanation that gives a reason for the administration’s choice of an amphitheater’s shape for the cinema. The administration would like to build a gas station between the three main roads, joining the three areas. What is the optimal point that is the same and shortest distance from the three roads? After it was built, a fire broke out in the gas station: there are two possible options for extinguishing it.

In the first option, the firefighters and the burning gas station are on the same side of the river. In the second option, the firefighters are on the opposite side of the river. Considering that the firefighters need the water from the river in order to extinguish the fire, which point on the river is optimal for shortening the distance between the fire and the firefighters?

A man, unaware of the previous events in the city, is building his house in one of these housing areas. He is interested in hanging a mirror of a certain size in the entrance, so that he can see himself completely in it when entering the house. What do the dimensions of the mirror have to be in order to satisfy the man's needs?

The following figure shows the structure of the content that includes both real-life geometry situations and abstracted geometric theorems:

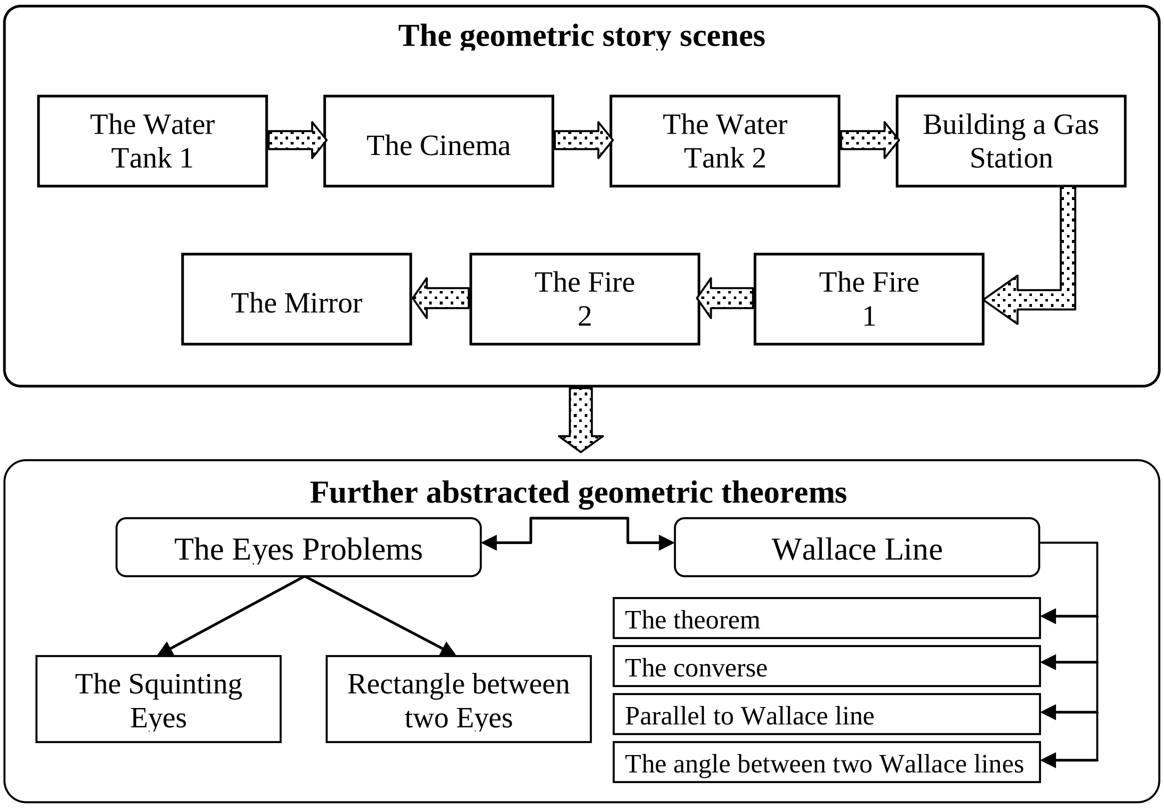


Figure 4: The structure of the suggested content.

As seen in Figure 4, every geometric situation throughout the whole story is a prerequisite for starting the next one, either due to the geometric concepts structure or to its sequence within the geometric story (see the accompany CD-ROM).

For instance, the first situation “the water tank problem 1” is required to be solved before entering the next situation because the learning outcomes of the first problem – the central angle, inscribed angle and the central angle theorem – are prerequisites for thinking about a geometrical explanation in the cinema problem. Likewise, the third problem “the water tank problem 2” depends on the first one in the story context as a building extension for the city.

All of the geometric content – concepts and theorems – in the seven real-life activities are prerequisites for studying the content of six geometric abstracted theorems, two activities named “the eye problem” and the other four activities classified under the title “Wallace line problem”, which includes the Wallace line theorem, the converse theorem, the parallel to Wallace line and the angle between two Wallace lines.

3.1.5 Prerequisites and Learning Outcomes

The following table shows details of the prerequisites and new learning competencies of every geometric activity, in both real-life situations and abstracted theorems. This analysis plays an important role in designing the geometric tests, which will be presented in detail in this chapter:

Table 2: Prerequisites and new learning competencies of the suggested content.

Activity	Lessons	Prerequisite competences	New competencies
The Water Tank 1	1. Building the water tank between the three housing areas	The perpendicular bisector and congruence of triangles.	Circumcircle and circumcenter of a triangle. The circumcenter of a triangle is the intersection point of the perpendicular bisectors on its sides. To define the circumcenter of a triangle and to construct its circumcircle.
	2. The converse	Congruence of triangles.	Perpendicular line from the center of a circle bisects the chord.

Activity	Lessons	Prerequisite competences	New competencies
	3. Corollary	The measure of the interior angles of a triangle.	The inscribed angle, the central angle theorem and Thales' theorem.
The Cinema	Building a cinema	The inscribed angle and the central angle. The central angle theorem.	The inscribed angle theorem.
The Water Tank 2	Extending the city with another housing area	The inscribed angle theorem. The measure of the interior angles of a triangle.	The cyclic quadrilateral. The sum of opposite angles equals 180°. To construct a cyclic quadrilateral.
The Gas Station	Building a gas station between the main three roads	The angle bisector and the congruence of triangles.	Inscribed circle of a triangle and the tangent line to one circle. The triangle's incenter. The intersection point of the three interior angle bisectors of a triangle is the center of the inscribed circle that touches the three. To define the incenter of a triangle and to construct its inscribed circle.
The Fire 1	On the same bank of the river	Reflection, congruence of triangles and the measure of opposite angles.	Solving the problem and justifying the optimal solution.
The Fire 2	On the other bank of the river	Transformation, alternate interior angles, corresponding angles and interior angles of a transversal.	Solving the problem and justifying the optimal solution.

Activity	Lessons	Prerequisite competences	New competencies
The Mirror	The optimal size of the mirror	Similarity of triangles and reflection.	Solving the problem and justifying the optimal solution.
The Eyes	1. Rectangle between two eyes	The tangent of a circle, inscribed angles theorem, Similarity of triangles and corresponding angles.	Solving the problem and justifying the optimal solutions.
	2. The squinting eyes	The tangent of a circle and Similarity of triangles.	
Wallace Line	1. The theorem	Vertically opposed angle, inscribed angles theorem, supplementary angles and the cyclic quadrilateral.	Proving the geometric statements.
	2. The converse	As previous.	
	3. Parallel to Wallace line	Supplementary angles, inscribed angles theorem and Thales' theorem.	
	4. The angle between two Wallace lines	Supplementary angles, the measure of opposite angles and Thales' theorem.	

3.1.6 Learning Phases

According to the review of prior studies in the field, there are many studies based on dynamic geometry that rely on separate individual real-life geometry activities, the present attempt presents a holistic approach, which introduces students to a geometric story that includes many of those real-life activities, whereas the power of the suggested approach, in the present study, is not limited to this view; it also smoothly evolves with students starting from a scene in the story in the first phase, then through experimenting, making conjectures, formulating geometric statements, getting proof's idea, justifying their conclusions and reaching the level of proof in the final phase.

This approach is not self-learning or a standalone learning environment without the role of a teacher; the teacher's guidance along the learning journey plays an important role in every phase.

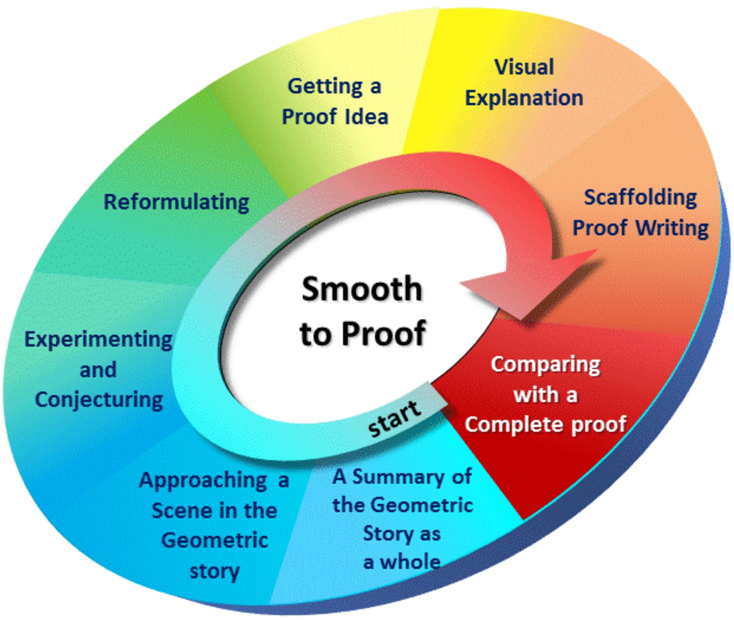


Figure 5: The sequence of the learning phases

The following shows these phases through one scene taken from the story as an example:

Phase 1: Approaching a real-life situation

In this phase students are able to discuss and realize a scene from the whole geometric story that requires doing geometry in order to set up the water tank so that it is exactly equidistant from all the three housing areas. The didactical aspect of this phase is to keep students motivated while they delve more deeply into geometric proprieties, concepts and theorems, so that they are still able to realize the relevancy of the concluded knowledge for the whole geometric story, and then wait to investigate the next scene and the included task.

Phase 2: Experimenting and conjecturing

After having discussed the situation, students moved on to the experimentation phase using dynamic geometry software, which could change students' attitudes toward geometry as not being just a professional task carried out only by geometers. Here each student paired up with his/her partner and they shared their conclusions. This phase was divided in the current example into three parts. In each part the students were encouraged to drag point D "the water tank" in order to determine the possible locations for the point that makes D equidistant from the three housing areas. The instructions provided to the students were as follows:

Experiment 1:

The water tank at point "D" should be the same distance from all three housing areas.

We begin with a simple task. First, you should find where the best point is that will place the tank in the middle between the two housing areas.

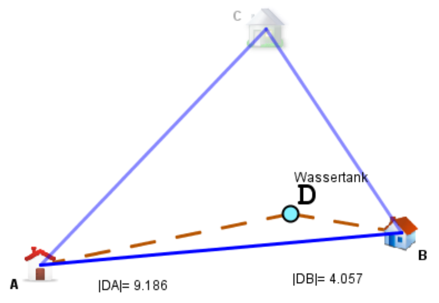


Figure 6: The first applet before moving the point D.

• Move the blue point so that the tank lies exactly in the midpoint between the two areas. Write down your observation in the given handout. Note: The blue point is a movable point.

• Compare the results with your class partner and then with the whole class.

Write down the possible solutions with configurations in the given handout.

Experiment 2:

There are many possible points that satisfy the condition that the tank lies equidistant from the two housing areas “A and B”; these points are collinear. Which point in your opinion is the optimal point if we also include the third housing area "C" in our calculations?

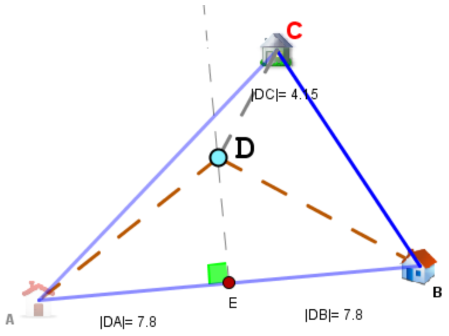


Figure 7: The second applet before moving the point D.

• Change D, so that AD = BD = CD. What do you think?

Write down your conjecture.

• Change the form of the triangle ABC and move point "**D**" again. Is your conjecture still correct?

Write down your conclusions with their configurations in your handout.

Experiment 3:

Is your conjecture still correct, so that the tank is equidistant from the two housing areas **A and C**? Does **D** also lie on the perpendicular bisector of **AC**?

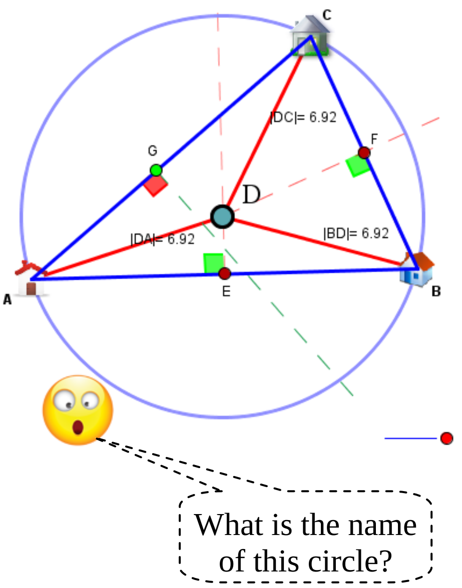


Figure 8: The third applet in the experimentation phase.

• Deform the triangle ABC; is your conjecture still correct?

• Investigate why the intersection point **"D"** of the three perpendicular bisectors is also a "Center".

"The green point G is a movable point"

Hint

Move the red point, which is in the bottom right corner of the figure to the right.

What is the name of the circle?

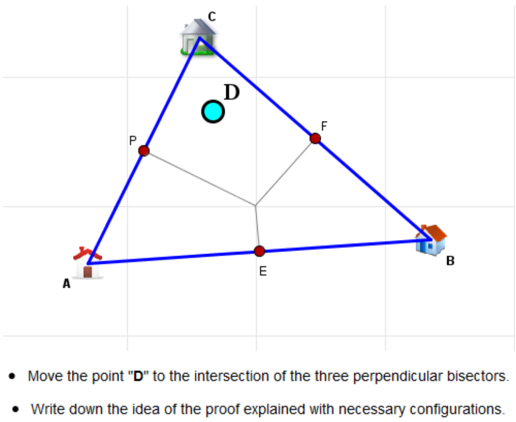
Note your conclusions with configurations.

Phase 3: Reformulating conclusions

In this phase all of the students were invited to participate in a classroom discussion. This phase might eliminate students' negative emotions if their observations or conclusions are different to those of other class members, while the teacher's questions and comments also play a vital part in giving feedback and unifying the students' responses. This phase could also facilitate a smooth transfer from students using their own language to adopt the use of the appropriate mathematical language. In addition, the teacher here should emphasize that there is not only one correct formulation for geometric concepts and theorems, as we can formulate geometric conclusions in several language structures, as long as the formulation includes the essential geometric proprieties. In this way, the teacher can stimulate students to play an active role in the geometry class, as they participate in formulating geometric definitions and theorems by themselves.

Phase 4: Getting a proof idea

In order to get a proof idea, dynamic geometry software with its dragging and measuring features was used in addition to the built-in programing language scripts that could provide an appropriate learning context to increase students' ability of visual recognition for the geometric constructions and illuminate the key ideas which will allow them to start proving.



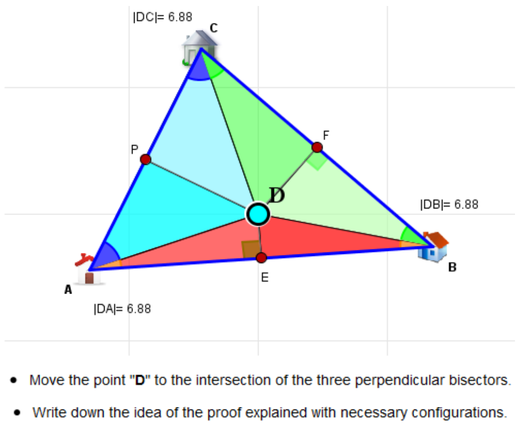


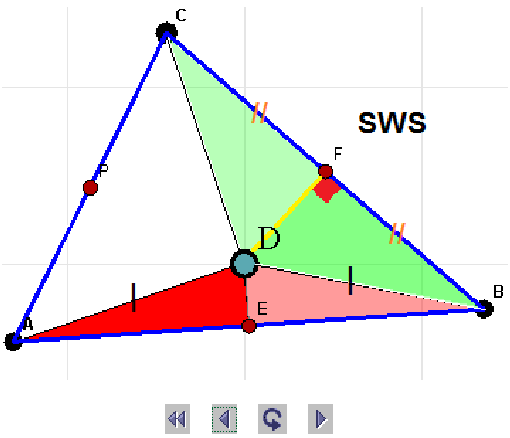
Figure 9: Proof's idea before dragging

Figure 10: Proof's idea after dragging

In this phase, and as Vygotsky stated, stimulating the “aha” reaction, or in other words a sudden insight, does not mean that students have discovered or deduced a new idea or relation by themselves (Vygotsky, L., 1978, p. 45). Nevertheless, this sudden insight and recognizing the key ideas could be recognized as the first step and a candle lighting the way ahead to justifying conjectures and writing the geometric proof.

Phase 5: Visual explanation

In this phase, students were invited to explain what they can visually conclude from each step in sequenced illustrations, and then to write this down in their own language. This phase also might make use of the constructivist learning term “scaffolding” as it depends on reducing the cognitive load and the difficulty of using mathematical terms, while allowing students to use their visual ability to formulate a geometric justification.



- Note each step in the given handout.
- Try to explain each step.
- Write down the proof by combining these steps.

Figure 11:Visual explanation.

Phase 6: Scaffolding proof writing

As shown in Figure 12 below, by making use of JavaScript and its image-hover effect, this phase is aimed at evolving students’ ability from the use of their own language to justifying their explanation, and scaffolding proof writing step by step. This was not a standalone self-learning phase as the teacher’s questioning and hints played an important part in guiding students from one conclusion to the next. The students were also invited to rewrite down each step in the provided worksheets (see Appendix H and N) using appropriate mathematical language.

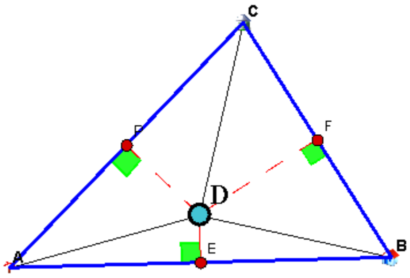
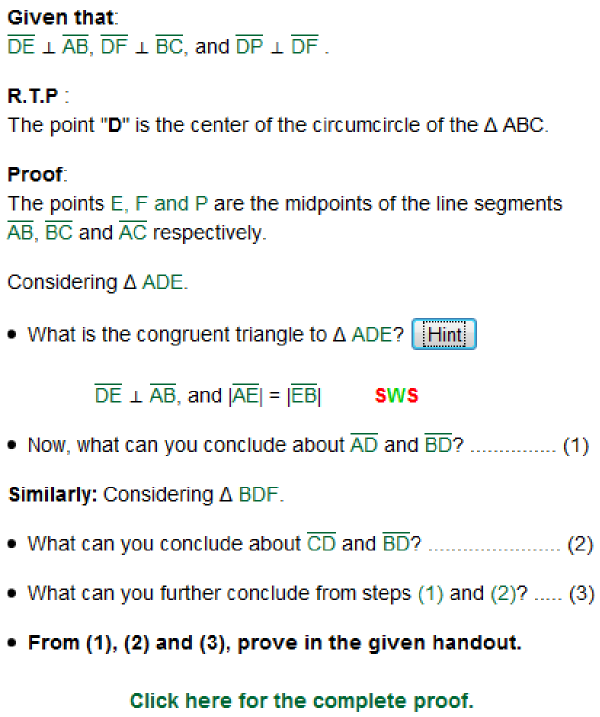


Figure 12: Hover effect pictures in stepwise proof scaffolding.



Given that:
$\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, and $\overline{DP} \perp \overline{DF}$.

R.T.P :
The point "**D**" is the center of the circumcircle of the Δ ABC.

Proof:
The points E, F and P are the midpoints of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively.

Considering Δ ADE.

- What is the congruent triangle to Δ ADE? Hint

$\overline{DE} \perp \overline{AB}$, and $|\overline{AE}| = |\overline{EB}|$ SWS

- Now, what can you conclude about $\overline{AD}$ and $\overline{BD}$? .... (1)

Similarly: Considering Δ BDF.

- What can you conclude about $\overline{CD}$ and $\overline{BD}$? .... (2)
- What can you further conclude from steps (1) and (2)? ..... (3)
- **From (1), (2) and (3), prove in the given handout.**

Click here for the complete proof.

Phase 7: Comparing with the complete proof

After having justified their explanations and rewritten them using better mathematical language than in the 5th phase, the students were invited to compare their proofs, which they wrote down in worksheets in phase 6, with the complete proof provided in a PDF file to check any mistakes, and improve their proof writing.

By reaching the end of one scene of the geometric story, it is expected that students will still be motivated to enter the next one, which could be an extension to the just-finished task or a new geometric situation that forms a logical sequence in the whole story.

These phases are still the same in terms of presenting the two abstracted geometric activities in the suggested content; the eye problem and Wallace line. However, the first phase which outlines the real-life situation was replaced with an explanation of the relations between the included activities in the geometric story and the abstracted geometric theorems, while the acquired geometric knowledge from the real-life activities played the role of a prerequisite for both activities "the eye problem and Wallace line".

3.1.7 Teachers' roles

Whereas the suggested approach is not one of self-learning environments, the teacher's role is essential for enabling students to move from one phase to another in every activity. So, the teacher's role as a facilitator for the whole learning process should be activated by:

1. Posing the problem of every geometrical situation to students.
2. Discussing the problem with the students.
3. Encouraging the students to experiment using the pre-constructed applets and observe the invariant properties in the geometrical configuration despite dragging one of its element.
4. Stimulating students' different formulations of their conclusions and conjectures, and emphasizing that they write them in their own language.
5. Giving feedback to all students' participations in an attempt to unify and use them as launching points for reformulating the geometrical statements or theorems in the formal mathematical terms.
6. Managing the discussions among students and encouraging students to pose questions relevant to the content of the geometrical situation.
7. Directing the students' attention to obtaining the key idea of the proof using the pre-scripted applet and concluding if-then relationships through dragging one defined element of the geometric configuration.
8. Encouraging the students to give geometrical explanations for pre-designed visualized steps, starting with the recognized proof idea and in their own words.
9. Scaffolding geometric proof writing by giving students appropriate hints and encouraging them to work collaboratively.

3.1.8 Students' roles

The students in the suggested approach have several roles that differ according to the phase of the geometric story that the student is in. Six different roles that the student should be encouraged to perform are listed below. Two roles "posing relevant questions and collaborating with class partners" will have the effect, along with each of the other four roles, of "experimenting, conjecturing, explaining and writing the geometric proof".

1. Experimenting

The first role that follows realizing every geometric situation, even in the abstracted geometry activities, is using the pre-constructed dynamic geometry applet to explore the proprieties of the geometric configuration in order to come up with various observations and conclusions.

2. Conjecturing

Based on the observations and conclusions in the previous phase, the second students' role is for them to write these down in their own language, and try to improve upon them during discussions and by using appropriate mathematical language.

3. Explaining

Here, the students have to look for the geometric explanations as to why their conclusions and conjectures are true in order to convince others, also with help of the pre-scripted applet that was designed to visually stimulate establishing of if-then statements.

4. Collaborating

The student's role here is not only to obtain or achieve the included geometric knowledge but also to share observations, think about results and reflect on the conclusions of his/her partner, in order to magnify increase the benefits of investigating the geometric situation, and to help his/her partner overcome learning difficulties.

5. Posing relevant questions

The benefit of this “questioning” behavior is not only important for the student who poses the question; it could be very effective for both the teacher and other students in the class as it gives the teacher the required view about how students think and in which direction, which enables the teacher to offer appropriate hints and use such relevant questions as launching points to state geometric concepts and even reform conclusions in appropriate mathematical language.

6. Writing proofs

Writing proofs is one of the geometric reasoning processes which might be considered the most difficult rule in geometry class. Also, the students here attempt to construct proofs with the help of a pre-planned dynamic worksheet using JavaScript to scaffold the formulation of the proof step by step. The following figure gives an overview of the geometry class environment in the suggested approach.

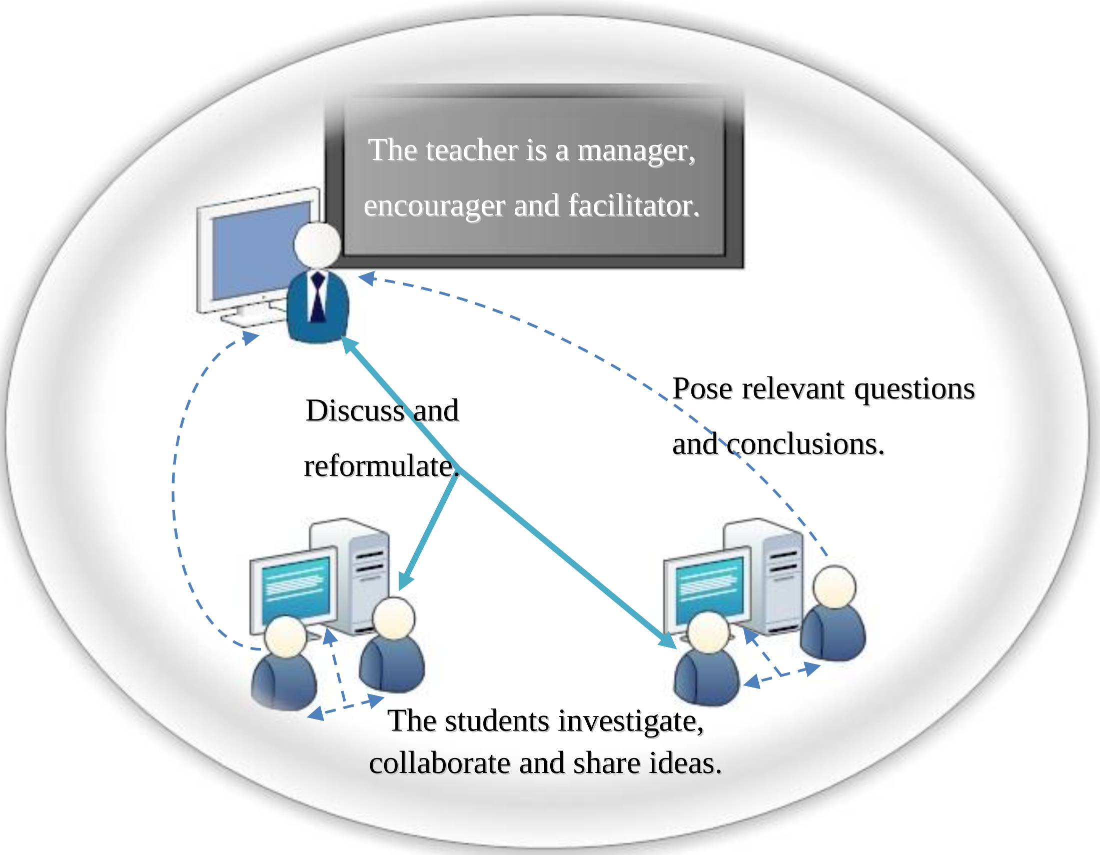


Figure 13: The geometry class in the suggested approach

3.2 Developing the Activities' CD-ROM

This part presents the standards for developing the CD-ROM, the programing languages and authoring software that were used in producing the activities in the CD-ROM. It also describes the basic features of the learning interface, the system requirements and also the procedures of checking the technical and the didactical appropriateness.

3.2.1 Standards for developing the CD-ROM

In order to develop the activities of the suggested approach and produce the learning environment, there are some of technical aspects that had to be considered. The following points show the important guidelines which are important for ensuring the usability and the suitability of the CD-ROM for achieving the aims of the suggested approach.

1. The learning interface for all of the whole activities is designed so that it does not require advanced computer skills for students to be able to use it.

2. Software such as "Flash Player, Java Virtual Machine and Adobe Acrobat Reader" was required before installing and using the activities. Thus, all these requirements were provided in the root directory of the CD-ROM.

3. The CD-ROM and all the activities were developed in order to fit different operating systems "e.g., different Windows versions, different Linux distributions and MAC OS" and to be compatible with different HTML browsers "e.g., Firefox, Internet Explorer, Google Chrome, and Safari".

4. The HTML pages and the embedded dynamic geometry Java applets were designed so as it avoids the need to scroll, and thus fit the 1024x768 pixels as the minimum screen resolution in today's computers.

5. It was also considered to design the HTML pages for the activities to open in full screen, because with the use of the simple navigation bar in the activities, the students do not need the default tool bar of the browser, which could save about 100 vertical pixels of screen resolution, and also the HTML pages were designed to be scalable for different sizes of screen resolution.

6. In some activities, to avoid scrolling, it was required to break the experimentation phase into two or three parts of experimentations in several pages.

7. According to one of the basic principles of designing a multimedia learning interface, it was considered to avoid the use of a colored background, decorated text or unnecessary elements such as pictures or animations.

8. In designing most of the activities and their learning phases, the interaction between student and content was considered through the use of the built-in scripts in the dynamic geometry software such as CindyScript in Cinderella, or through passing conditions to the dynamic geometry applets using JavaScript.

9. It was considered in designing the dynamic geometry applets to have an appropriate size to enable students to use it and at the same time to fit the screen with the text for instructions, explanations, and questions.

10. It was also considered in designing the dynamic geometry applets, to make the labels of geometric elements such as points and lines to be clear and not overlapping on another.

11. It was also considered to formulate the tasks using conversational style and avoid the use of long static texts.

12. It was considered to provide some hints to scaffold explorations and justifications in the learning phases as it was expected that students, who have negative attitudes, might need further help and explanation.

13. The above further help and explanations were considered to be provided in the style of hypertext, so that students can click on it only when they need such support.

3.2.2 Programing languages and software

This section presents some examples from programing language scripts and authoring software that were used in developing the activities of the suggested approach and the CD-ROM. Two of different dynamic geometry software packages were used in designing the embedded applets in the activities, namely GEONExT and Cinderella. The built-in scripts in both programs played an important role in constructing an interactive and dynamic geometry learning environment. The following figure shows some of those scripts that were used in GEONExT applets.

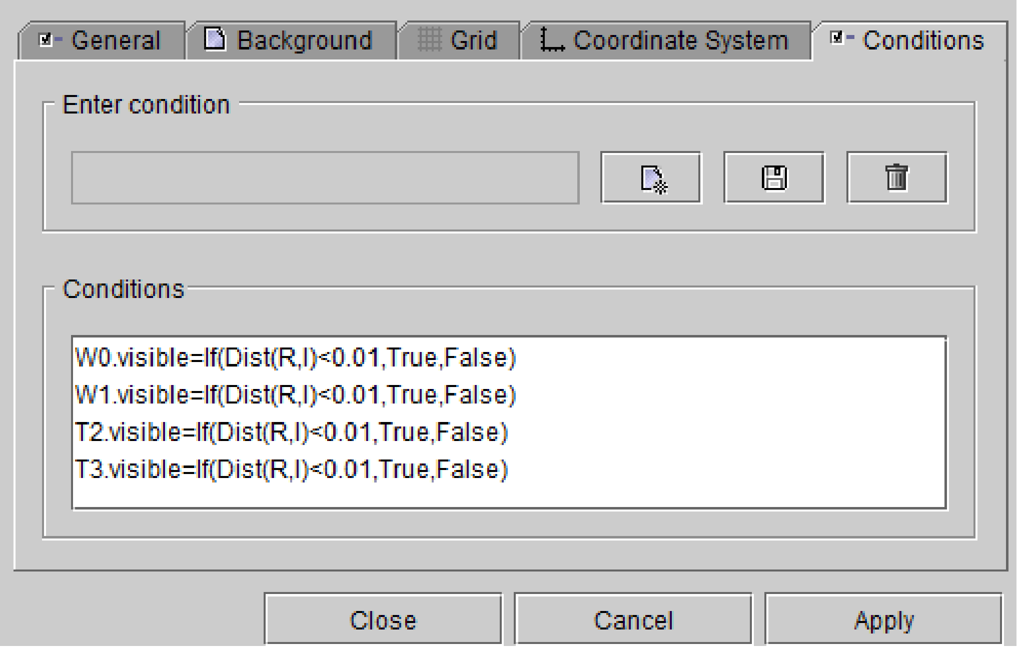


Figure 14: GEONExT scripts that control the visibility of geometric elements

The above script shows the condition that makes one geometric element, such as the two angles W0 and W1, the texts T2 and T3, visible or not according to the distance between the two points R and I that should be shorter than 0.01. In this way the task is to drag point R so that it is congruent with point I “the intersection between the chord and the circle”, so the expected conjecture is that the line AR is parallel to the Wallace line which passes through the three points F, G and H as a conclusion of the two opposite angles ∠ (GAR) and ∠ (AGF).

Also, it is clear from the conditions window above that the expression ‘visible’ is the method for showing or hiding the targeted geometric element with the dependent values ‘True’ or ‘False’ which operated in ‘Show’ or ‘Hide’, respectively. The following figure shows the effect of the above scripts before and after dragging point R into point I.

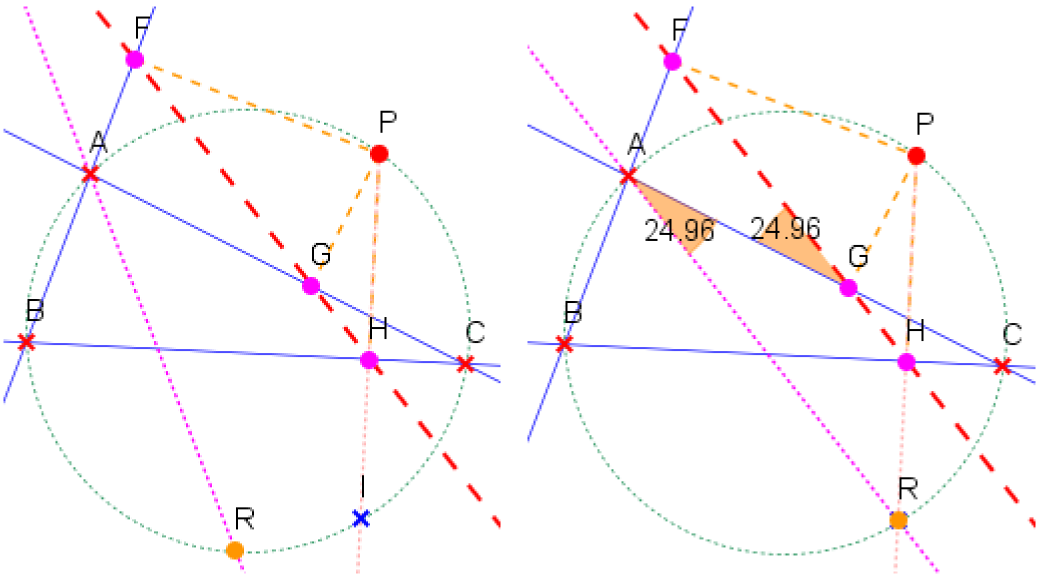


Figure 15: The effects of the scripts after dragging point R in GEONExT

The next figure shows a similar example of the same method of controlling the appearance of the geometric element in Cinderella, using the built-in script named CindyScript. The expression here is a little different; here the method’s expression is “isshowing” and it describes the condition of defining when, for example, polygon2 is shown. Also, if the distance between the two points P1 and U is shorter than 0.3, the first value ‘True’ will be operated, otherwise polygon2 will disappear.

#	Operation	Result
0	Poly2.isshowing=if(dist(P1,U)<0.3,true,false)	false
1	Poly3.isshowing=if(dist(P1,U)<0.3,true,false)	false
2	Poly4.isshowing=if(dist(P1,U)<0.3,true,false)	false
3	Poly5.isshowing=if(dist(P1,U)<0.3,true,false)	false
4	Poly6.isshowing=if(dist(P1,U)<0.3,true,false)	false
5	Poly7.isshowing=if(dist(P1,U)<0.3,true,false)	false
6	Text1.isshowing=if(dist(P1,U)<0.3,true,false)	false
7	Text3.isshowing=if(dist(P1,U)<0.3,true,false)	false
8	Text0.isshowing=if(dist(P1,U)<0.3,true,false)	false
9	E2.isshowing=if(dist(P1,U)<0.3,true,false)	false
10	E3.isshowing=if(dist(P1,U)<0.3,true,false)	false

Figure 16: The CindyScript programing language

The label ‘D’ refers to point P1 in the above scripts and point U is the intersection point of the three perpendicular bisectors.

These scripts played a vital role in enabling students to recognize the congruency of three pairs of triangles, through dragging point D into the intersection of the three perpendicular bisectors, and getting the idea of justifying their conclusions and formulating the geometric proof.

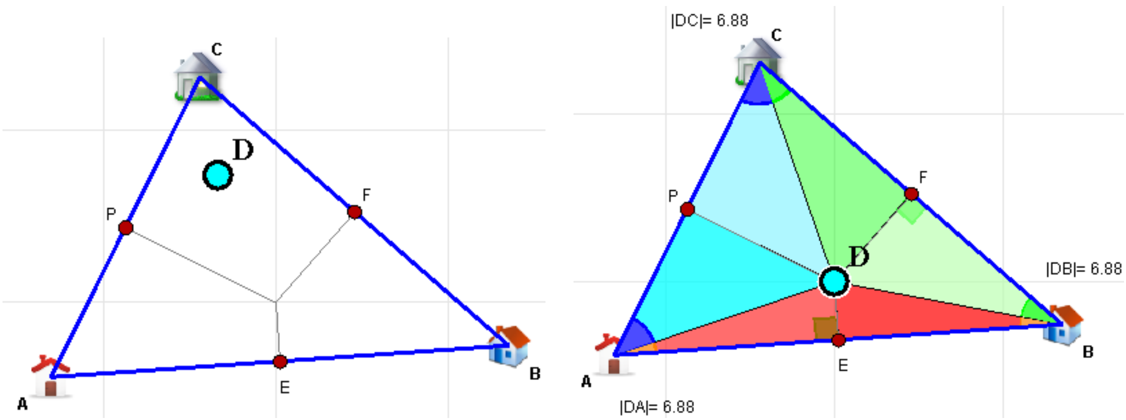


Figure 17: The effects of CindyScript before and after dragging

Another method was used in designing the geometric applets in both GEONExT and Cinderella, which was changing the colors of the geometric elements, which could emphasize a geometric relationship between the elements of the geometric configuration if a specific condition occurred through dragging another element to a predefined position. The following figure shows an example of the mentioned method using CindyScript.

4	g.isshowing=if(dist(C,G)<0.2, false, true)	true
5	E0.color=if(dist(C,G)<0.05, [1,0,0], [1,1,1])	[1,1,1]
6	E1.color=if(dist(C,G)<0.05, [1,0,0], [1,1,1])	[1,1,1]
7		

Figure 18: The CindyScript that controls the colors of elements

As shown in the script, the expression ‘color’ is used instead of ‘isshown’ in the previous script to control the color of the element ‘E0’, for example, if the distance between the two points C and G is shorter than ‘0.05’ the color will be red as the code of ‘RGB’ shows the value [1, 0, 0], otherwise the filling color will be white [1, 1, 1].

The dynamic geometry software also has many features for producing several interactive and dynamic effects that could be valuable in modeling the real-life geometry situation, for instance in Cinderella it is possible to make a multilayered applet (see, for example, the mirror problem in the accompanying CD-ROM). The following figure shows the window for controlling the properties of geometric elements; under the title ‘Render Options’ there is a method for defining the plane to be ‘the level of the layer’ of every geometric element. This method was used in showing and hiding the picture of the man’s head or the man’s leg in the mirror using a white layer over the picture, which also depended on the proprieties of the reflection, and through dragging the mirror vertically using the blue slider point.

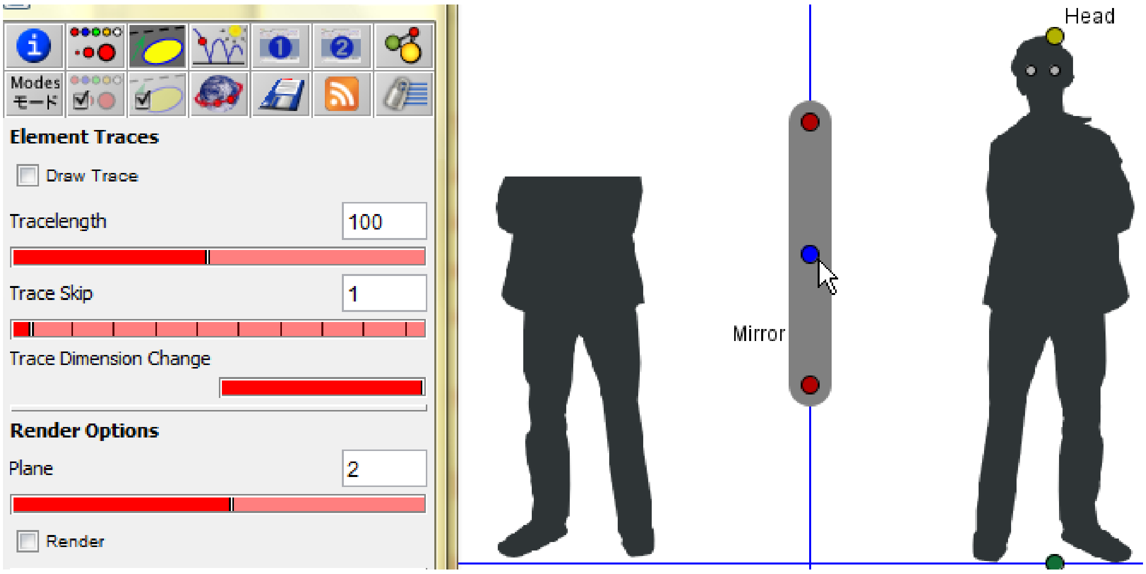


Figure 19: The window of changing the plane of geometric elements and its effect

Another method of sending and controlling the applets from the HTML page is dynamic text or buttons using JavaScript. The following script is an example from one of the dynamic geometry activities that were developed using GEONExT.

* klicke hier*

This script is also a method for changing the status of the visibility of several elements simultaneously (see the following figure) by dragging and pinning point R into point I with just single clicks. After clicking, the students are invited to move point P onto the circle, so that they will be able to generalize the collar of the parallel line to the Wallace line.

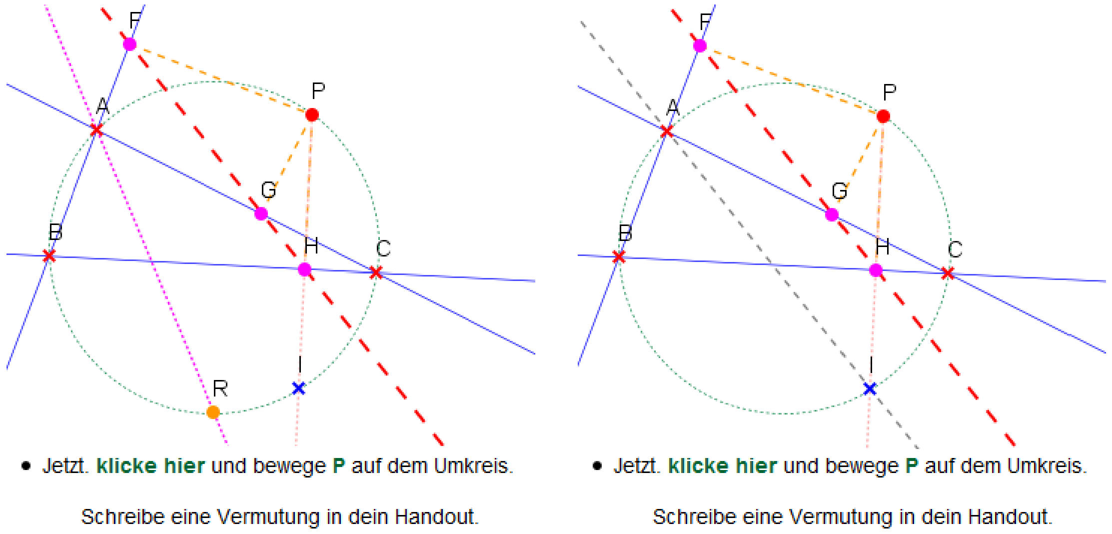


Figure 20: Point R unites with point I after clicking

Some other authoring software was used in designing and adapting the scripts and syntaxes of the dynamic geometry activities pages such as the PSPad editor, which was used in editing HTML and JavaScript.

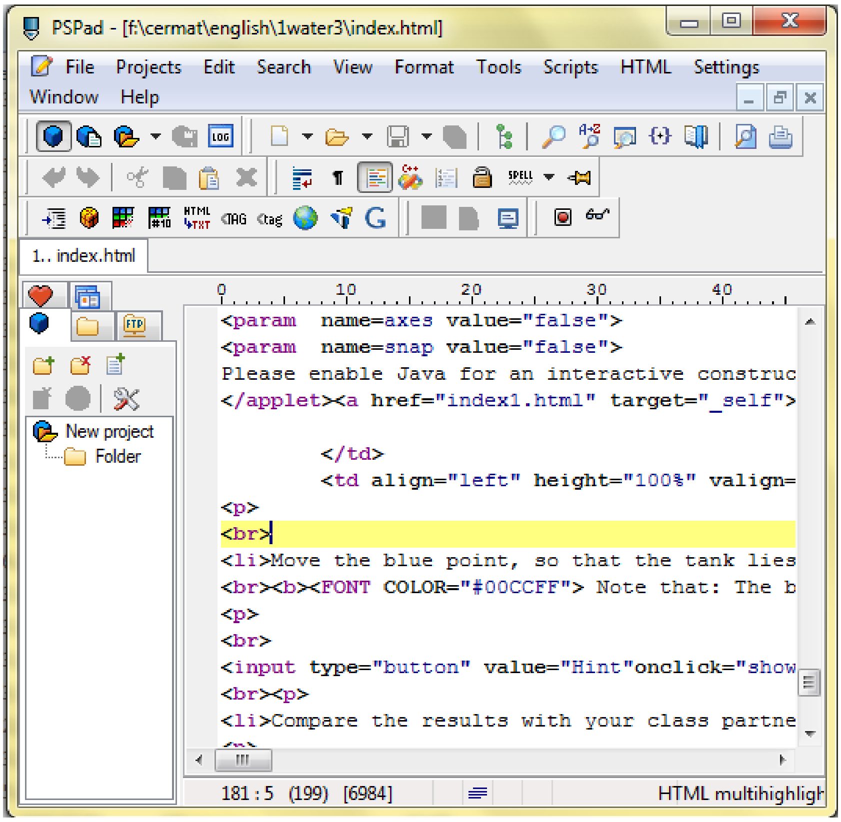


Figure 21: PSPad the freeware programming editor from Microsoft

A well-known multimedia authoring program, Director 8, an earlier version of Adobe Director and its internal programming script ‘Lingo’ were used in producing an introduction flash movie for all of the situations in the geometric story.

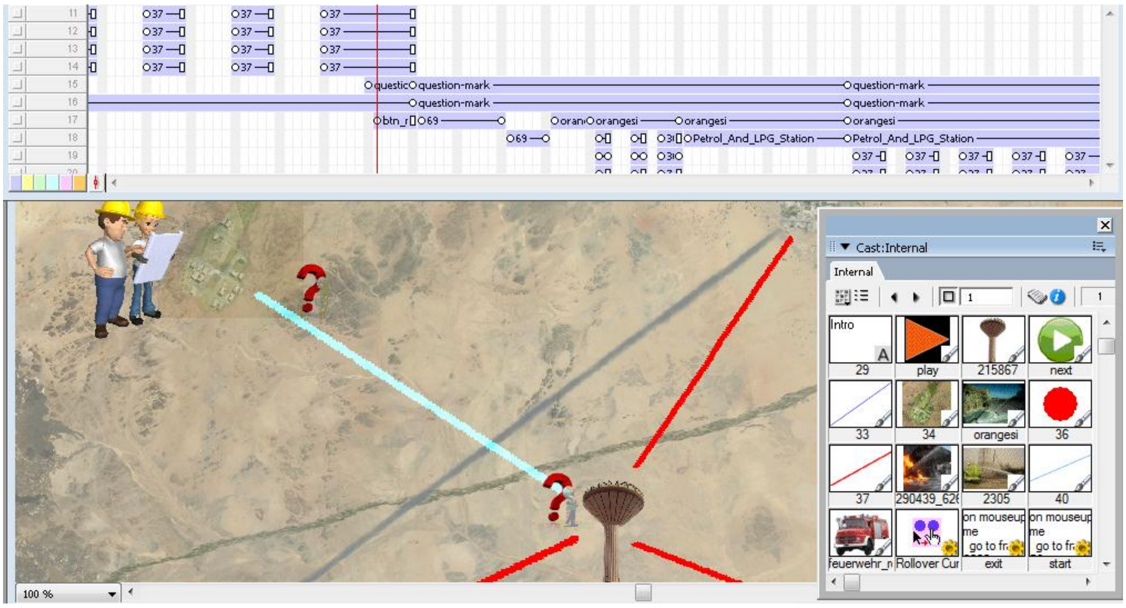


Figure 22: A screenshot from the project score, stage and the cast members in Director

3.2.3 Learning Interface

The learning interface started with a flash movie as a summary of the whole geometric story followed by the list of the geometric activities.

Figure 23: The flash movie of the entire geometric story

The basic design of the navigation bar was adapted from dynamic worksheet creator which is used in integrating dynamic applets into the HTML instructions pages, and using the PSPad editor. Also, the title of every learning phase was located between the next button on the right and the previous button on the left. In the last page ‘the proof phase’ of every activity, the next button navigates back to the list of the activities; likewise the previous button has the same function at the beginning of every activity.

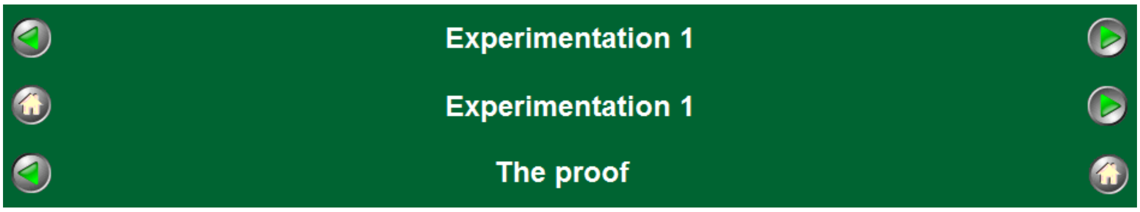


Figure 24: The basic design of navigation

As shown in figure 25, the rest of the page is divided into two main parts: the first part on the left includes the introduction to the geometric situation ‘1’ and the geometric configuration ‘2’, while the second part on the right side includes instructions, hints, questions and tasks ‘3’. In the first activity there was a link to a flash movie ‘4’ that illustrates how the dynamic geometry configuration applet could be used. Furthermore, the activity pages were designed to be scalable and resizable according to the user’s screen resolution and to fit the contained texts that differ from one phase to the next.

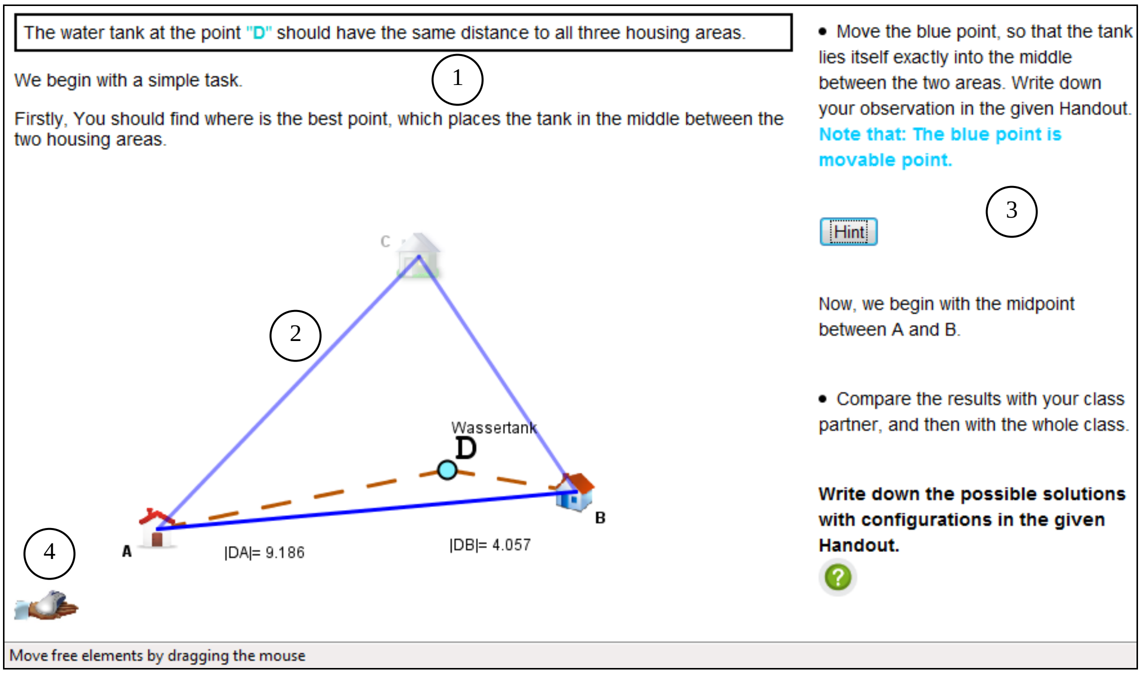


Figure 25: The design of the dynamic activity page

3.2.4 System Requirements

The developed CD-ROM is compatible with the following minimum computer specifications: 128 megabytes of RAM, resolution starting from 1024x768, and a sound card. The CD-ROM was also packaged to work on several platforms and could be installed on several operating systems such as Microsoft Windows (XP/Vista/7), Mac OS and the different distributions of Linux. Other conditions are required for installing this learning software; namely Java virtual machine, Adobe Acrobat reader and Flash Media Player. As the learning software requires the use of a web browser, it was made to be compatible with most of today's web browsers. However, due to some technical difficulties and limitations, Firefox or Safari is recommended.

3.2.5 Preliminary Evaluation of the Suggested Approach

With the intention of ensuring the appropriateness of the suggested approach and its activities to achieve the purposes of the present study, the activities were presented to a group of experts in teaching and learning mathematics in Egypt and Germany through several seminar sessions and individual interviews. The participants in these seminars and interviews were asked to discuss and suggest important modifications within the following guidelines:

1. The suitability of the suggested approach and its organization for the students who have negative attitudes towards geometry and might also have several learning difficulties.

2. The suitability of the included activities to improving students' attitudes and overcoming their learning difficulties.

3. The appropriateness of using dynamic geometry facilities to facilitate the learning activities included in the suggested approach.

4. The appropriateness of the instructions and hints that were provided in every activity for guiding both the teacher and the students during all of the learning phases.

5. The clarity of the mathematical terms and the language used in preparing the geometric activities.

6. The ease of use of the interface of the entire learning environment.

7. The compatibility of the CD-ROM and the included software components with the different operating systems and browsers.

According to the experts' recommendations and seminar discussions, two activities were removed, that might be in higher level than the most targeted students have, and although they were two real-life applications in the field of trigonometry rather than geometry. In addition, as the Egyptian students and even teachers had no experience with dynamic geometry software, an auto-run flash movie was embedded instead of the Java applet, in the first geometric task, in order to illustrate how the dynamic geometry applet could be used.

3.3 Instrumentation

According to the purposes of the present study, there was a need to garner a clear view of students' attitudes towards geometry, towards using geometry in learning, and towards the suggested approach itself in order to ascertain the degree of effectiveness of the included activities in improving students' attitudes. In addition, a geometry entrance test was essential in order to determine whether the students have the minimum prerequisites for participating in the study experiments. Furthermore, and in order to open up the possibility to replicate the study or extend it, one more achievement test was designed. This included the new learning competencies that students might develop from studying the activities of the suggested approach, according to the predefined list mentioned earlier in the current chapter. Thus, the measurement tools of the present study are three questionnaires of attitudes and two achievement tests; one plays the role of testing the existence of the learning prerequisites and the other of measuring the geometric achievement may take place in study replication.

3.3.1 Geometric Tests

According Table 2 that showed the learning prerequisites and the new learning competencies of every activity, this part shows the processes and standards of designing both, the entrance and achievement tests. As most of the test items are dependent on the multiple-choice type, which are more effective than the true-false type and more objective than the essay questions, the following principles were taken into account:

1. In multiple-choice items double-negative forming was avoided, because it might cause confusion.

2. Complex multiple-choice questions known as type-K items, which present a combination of alternative answers, were avoided.

3. Each item was written clearly and objectively.

4. Including most of the item’s content in the stem, so that repeated or irrelevant text in the alternatives was avoided.

5. Keeping the alternatives parallel in form and avoiding clues that could be included in the stem or in the alternative answers such as grammatical formulation.

6. Avoiding copying and pasting from textbooks, because students may be familiar with a certain question, and accordingly their tasks will be just to recognize the text rather than understand it.

7. Avoiding the problem of splitting one item over two pages.

8. Avoiding the use of unnecessarily difficult vocabulary. (Burton et al., 1991)

9. Essay questions in the entrance and the achievement tests should be graded manually and separately one at a time.

Also, with the purpose of setting the final version of the geometric tests (viz., Appendix F and G), five important processes were considered:

3.3.1.1 Defining the content of the tests

Regarding the analysis of the content of the activities, the entrance test is concerned with measuring students' understanding of the following prerequisites: The perpendicular bisector, reflection, transformation, congruency of triangles, similarity of triangles, the angle bisector, the inscribed angle, the central angle, the measure of the interior angles of a triangle, the central angle theorem, the inscribed angle theorem, the measure of opposite angles, alternate interior angles, corresponding angles, interior angles of a transversal, the tangent of a circle, the supplementary angles, the cyclic quadrilateral and Thales' theorem. On the other hand, the achievement test was intended to measure whether the students had mastered geometric concepts and theorems such as the circumcircle and the circumcenter of a triangle, the triangle's inscribed circle and the incenter, the central angle theorem and Thales' theorem, the inscribed angle theorem, the cyclic quadrilateral and its proprieties, in addition to reasoning and proving skills.

3.3.1.2 Stating the grading method

All of the items in entrance test were multiple-choice items, so the student's expected response is to choose one of the alternatives in each item; the grading method was to give the right answer the mark “1” and the wrong answer the mark “0”. The same grading method was used in the achievement test for the multiple-choice items; every essay-type item in the achievement test was suggested to be graded with “5” marks, the five marks was distributed; one mark for defining the given information, and another one mark for defining or translating what required to prove into a provable mathematical form, and three marks for proof writing and configurations.

3.3.1.3 Judging and piloting the tests

Judging and piloting the tests was aimed at examining the readability and the clarity of each item, especially for the German students while the first version of the test was designed in Arabic and administered to 14 Egyptian students in the Faculty of Education at Suez Canal University. In addition, the appropriateness of the items for measuring the targeted geometric concepts and statements was discussed with experts and teachers of mathematics in Egypt and Germany.

As a result of these discussions, some of the items were reformulated to avoid the usage of difficult or inappropriate formulation or double-negative formulations, and the “type K” multiple-choice items were completely reformulated into an individual multiple-choice one. In addition, the mathematical and geometrical symbols used in the test were replaced in the German version with those that German students are used to (cf. the English version: Appendix F and G, vs. the German version: Appendix L and M). The Spearman-Brown coefficient of reliability was calculated for the entrance test using ‘SPSS’ in order to define to how efficient the test is at giving the correct and the same results through different times of measurements. The reliability coefficient was 0.707, which is considered as above-average reliability. Taking the square root of the reliability coefficient, we get the experimental validity of the test to be (0.707)2 = 0.499, which shows that the test has an experimental validity in the range between low and intermediate validity. Further experiments and piloting for the achievement test are required in order to calculate the reliability in future replication of the present study. Also, the present researcher suggests the use of the alpha coefficient instead of the Spearman-Brown coefficient to calculate the reliability for the achievement test, as it includes a different grading system for both multiple-choice and essay items. However, the achievement test was also presented to a group of teachers and experts in mathematics education to solicit their views as to the appropriateness of item formulations in Egypt and Germany. Accordingly, some items were removed or revised in order to assure clarity and appropriate formulation.

3.3.1.4 Defining the required time

The required time for completing the entrance test was estimated at about 30 minutes, as a result of taking the mean of the first student who finished the test and answered all the items, and the last student who finished the test and answered all of the items, as shown by the equation (minimum time required + maximum time required) / 2 = (45 + 20) / 2 ≈ 30 minutes. The same procedure was followed in determining the required time for the achievement test through administering to two Egyptian students and two German students; the estimated required time was 45 minutes.

3.3.1.5 Considering ethical issues

At the beginning of every test, the researcher acknowledged and stated that any personal information obtained from the test would be confidential. And to enable students to feel more confident, every student was able to write his/her name or just a code if he/she did not want to disclose it. This might put students into a more comfortable emotional state while completing the tests.

3.3.2 Questionnaires of Attitudes

Two questionnaires of attitudes were designed with the purpose of obtaining a view of the students' attitudes towards geometry, geometric proof and towards the use of computers in learning before the study experiments. In addition to administering the questionnaire of attitudes towards geometry and geometric proof again after the study experiments, another questionnaire of attitudes was designed to gather information about students' attitudes towards the suggested approach as well as its effectiveness in improving their negative emotions and false beliefs about geometry and geometric proof after participating in the activities of the suggested approach. All three questionnaires of attitude pass through the same procedures of formulating and judging the included items. However, every questionnaire has a different aim and different content, in order to measure several components of students' attitudes, e.g., their emotions, beliefs and behavior, particularly in the geometry class (See Appendix C, D and E).

3.3.2.1 Defining the aim and the content of every questionnaire

Also, the first questionnaire was concerned with gathering information about students' emotions towards geometry and geometric proof, their beliefs about the importance of geometry and the functions of geometric proof, in addition to their behaviors "participations" in the geometry class. The second questionnaire was aimed at obtaining a view of students' attitudes about using computers in mathematics learning and especially their experiences with dynamic geometry software. Whereas the third questionnaire was designed to examine attitudes towards the suggested approach as well as the effects of the included geometric activities in overcoming some geometry-learning difficulties and students' attitudes towards geometry and geometric proof.

Similar to the achievement tests, the students have the possibility to write their names or the same anonymous code that was used in the achievement tests.

3.3.2.2 Stating the type of the questionnaires' items

According to the review of literature and related studies, the Likert-type questionnaire was accepted in the context of the present study. Furthermore, several interviews with students and mathematics teachers in Egypt and Germany were conducted, as well as reviewing several questionnaires that were used in the prior related studies; the aim was to garner the common students' and even teachers' perceptions and learning difficulties in the field of geometry and geometric proof learning. Additionally, the clarity of items' formulations was considered, and numerous items were formulated, as it is expected that some will be revised or removed completely. Some of questionnaires' items were formulated negatively; these negative items were required to inverse the ranks system during entering and processing of the collected data in SPSS.

3.3.2.3 Judging and piloting the questionnaires

Three versions of the questionnaires were formulated in Arabic, English and German in order to present the included items to experts and mathematics teachers from both Egypt and Germany to allow them to judge the clarity of the formulations and the appropriateness of the included content, especially for the German students. Also, the preliminary version was administered to 14 Egyptian students in the Faculty of Education at Suez Canal University. Accordingly, some changes were made to avoid inappropriate formulations or misunderstanding. One of the most relevant and significant changes was in the type of response in the questionnaire items. In the Likert-type questionnaires there is a response which addresses the hesitating emotion in deciding the degree of the agreement or disagreement; this response normally takes rank "2" in the scale from "0" to "4", or rank "3" in the scale from "1" to "5". This type of response was adapted into a scale from "0" to "4" where "0" presents the student's response "I do not know" to express that the students have not passed through the experience or the content of the item before participating in the questionnaire, and instead of taking the value "2" or "3" it was recommended to take the value "0" to fit in with the context of the questionnaires, while the response "I do not know" does not express any degree of attitude – “Emotions, Beliefs, or Behaviors” – towards the content of any item. Moreover, the importance of this change was also obvious in the phase of the statistical analyses in order to avoid the possible inflations in the mean ranks from the students’ responses under “I do not know” if it was decoded and entered in the SPSS as value “2” or “3”.

3.4 Sampling

Traditionally, this part is concerned with the method of choosing the participants for the main study experiments. In fact, and according to the context of the present study which was created to deal with students who have learning difficulties and negative attitudes towards geometry and geometric proof, particularly those who will be mathematics teachers in the future after graduating in the faculties of education, the subjects of the present study were allowed to participate in its experimental procedures according to three criteria:

1. The participants should have to some extent the prerequisites of studying the content of the suggested approach, according to the results of the entrance test.

2. The participants in the sessions of the suggested approach should really have learning difficulties and negative attitudes towards geometry and geometric proof, according to the results of the questionnaire that was administered before the main study experiments.

3. The participants should agree to attend the main study experiments and study the content of the suggested approach.

The outcome of the above criteria was discernible in the sample size. From 20 Egyptian students in the mathematics section only 12 students were available for the study experiments and the pretest phase, and the drop-out rate in the German students group led to limiting the result of the present study to the Egyptian students group, as from about 200 students in the university of education Karlsruhe who participated in the pretest phase, only 6 students participated in the study experiments and only one student attended the post-test phase. Accordingly, the results of the present study will be limited to the group of Egyptian students.

3.5 Main Study Experiments

After considering the judgments of the experts in the field of mathematics education in Egypt and Germany, the final version of the suggested approach and its activities (see the accompanying CD-ROM) were administrated after the pretest phase to the sample of the study 12 pre-service Egyptian student teachers of mathematics in the winter semester in the academic year 2009, and 6 students in the university of education Karlsruhe in the summer semester of the academic year 2010. According to the limited time available for students to participate in voluntary activities aside from their studying for other courses, only six activities were presented to the Egyptian students group, and four activities to the German students group. The following table shows the number of sessions in every activity and the required time for studying the whole suggested approach.

Table 3: The required time for studying the content of the suggested approach

The geometrical situation	Sessions	Time/min
Introduction for the whole geometric story	1	5
Building the water tank between the three housing areas	3	135
Building a cinema	1	45
Extending the city with another housing area	1	45
Building a gas station between the main three roads	1	45
A fire broke out in the gas station	2	90
A man interested in hanging a mirror	1	45
The eye problem	2	90
Wallace line and related statements and properties	4	180
The sessions and required time for the entire activities	16	680 min 11hours and 25minutes

Also, before starting the sessions of the main study experiments, the researcher tested the computers' ROM and the installed operating systems to verify their readiness for installing the activities CD-ROM, as it was required to setup some programs such as

Java virtual machine, Adobe flash player and one of the recommended browsers Mozilla Firefox or Safari. In addition, the researcher compiled a student version of the CD-ROM in which the introduction was provided only in text form and did not include the voice in the flash movie, plus a "Teacher Version" presentation that included the voice instruction. This was aimed to avoid the noise and distraction that might arise if every student listened to the voice introduction in the flash movie at the same time. In addition, the students were asked to write all their observations, conjectures, conclusions and proofs in pre-structured worksheets (see Appendix H and N). The students were encouraged to learn together through discussion and sharing their thoughts, as students worked on one computer in pairs. In addition, the role of the teacher, as mentioned before, was mainly as a facilitator who provided the appropriate hints at the right time and also if the students asked for help, besides also playing the role of manager for the class discussions.

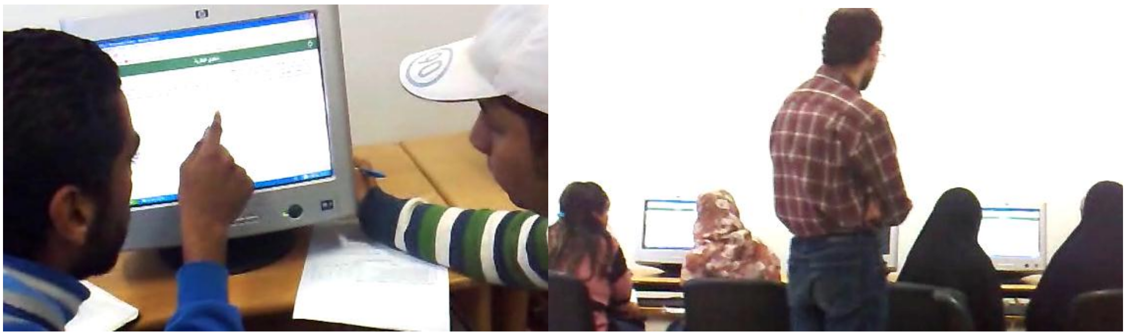


Figure 26: Two pictures from the study experiments in Egypt

During the sessions of the study experiment, some of students who had not attended the pretest phase, came and expressed their willingness to participate in the activities of the suggested approach. That was a result of having heard of some of the advantages of the activities from their colleagues who were participating in the main study experiments. After studying the selected activities, the students were again asked to fill out the questionnaire of attitudes towards geometry and geometric proof, in addition to the questionnaire of attitudes towards the suggested approach and its benefits. Afterwards, the researcher invited the students to participate in an interview to discuss the advantages of the suggested approach and its activities; the most affirmative argument during the interview was one student's question as to why such a geometry learning method does not currently exist in mathematics classes in schools and even in university.

Later, after discussing the phases and the activities of the suggested approach with the members of the Curricula and Instruction Department in two seminar sessions, both the dean of the Faculty of Education in Suez Canal University, and the dean of the department who is also a professor of mathematics instruction, expressed the importance of the included activities as a new trend in teaching geometry starting from the real situations and particularly with the use of dynamic geometry software, and recommended establishing a mathematics laboratory in the Faculty of Education that would aim to develop interactive learning environments in order to evolve students' attitudes and performance in mathematics.

On the other hand, and beside their low scores in the entrance geometry test, the German students were not very interested in participating in the study experiments, and some of them expressed that they did not wish to attend a mathematics class if there was no examination at the end of it; in other words, if it did not affect their marks. The present researcher considers this characteristic as evidence of their negative attitudes towards mathematics, and geometry in particular, while the students in the faculties of education are expected to attend mathematics classes, not to pass a final exam, but to broaden their mathematical knowledge as future teachers.

4 Results

In an attempt to examine whether there are significant differences in students' attitudes towards geometry and geometric proof, as a result of using the suggested approach. On the other hand, whether there are significant differences in their attitudes towards the suggested approach compared with their attitudes towards using computers in mathematics learning which were measured before intervention. The results in this chapter will be reported in four parts in order to answer the following questions:

1. What are students' attitudes towards geometry and geometric proof?
2. What are students' background and attitudes toward using computers in mathematics and particularly in geometry learning?
3. What is the effectiveness of the suggested approach in improving students' attitudes towards geometry and geometric proof?
4. What are the students' attitudes towards the suggested approach and its content?

The answers to the first two questions will deal with both groups; the group of Egyptian students at the Faculty of Education "Mathematics section" in Suez Canal University, and the group of German students at the University of Education Karlsruhe. Due to the drop-out rate in the group of German students, the answers to questions three and four will be limited to the group of Egyptian students.

4.1 Students' attitudes towards geometry and geometric proof

This part answers the first question according to students' responses "in both the Egyptian group and the German group before the experiments" on 20 Likert-type items of the questionnaire of attitudes towards geometry and geometric proof, in addition to their responses on an open question that enables them to give further comments. The variable attitude in the present study reflects not only students' emotions but also deals with their beliefs and behaviors in regard to geometry class. In the following there is a review of the most significant of the students' responses, which might provide a detailed view of the profiles of students' emotions and beliefs in regard to geometry class, while the second part summarizes students' comments in the open question about their learning experiences with geometry and geometric proof.

4.1.1 Profile of students' attitudes towards geometry and geometric proof

The fifth item was "*Proof is extremely hard so that it takes the enjoyment of learning away*." The mean ranks of Egyptian and German groups show discrepancy in the students' view of to what extent they see themselves able to do geometric proof and how it eliminates the enjoyment of learning. The mean of Egyptian students' response to the question was "1.75" which falls between "Strongly agree" and "Agree", while the mean of German students' responses is "2.75" which tends towards "Disagree".

Table 4: Mean ranks of item five "Difficulty of geometric proof"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
Proof is extremely hard so that it takes the enjoyment of learning away.	1.7500	1.11803	2.7500	1.33898

The following figure shows that the most common responses for German students were "Disagree" and "Strongly disagree", while Egyptian students' responses fell mostly into "Strongly agree" which might reflect that Egyptian students have more difficulties and negative attitudes than German students. In contrast, the figure shows that the German students' responses distributed to some extent on a normal curve and accumulated in the positive side; which gives one of German students' characteristics of better attitudes towards geometric proof than Egyptian students.

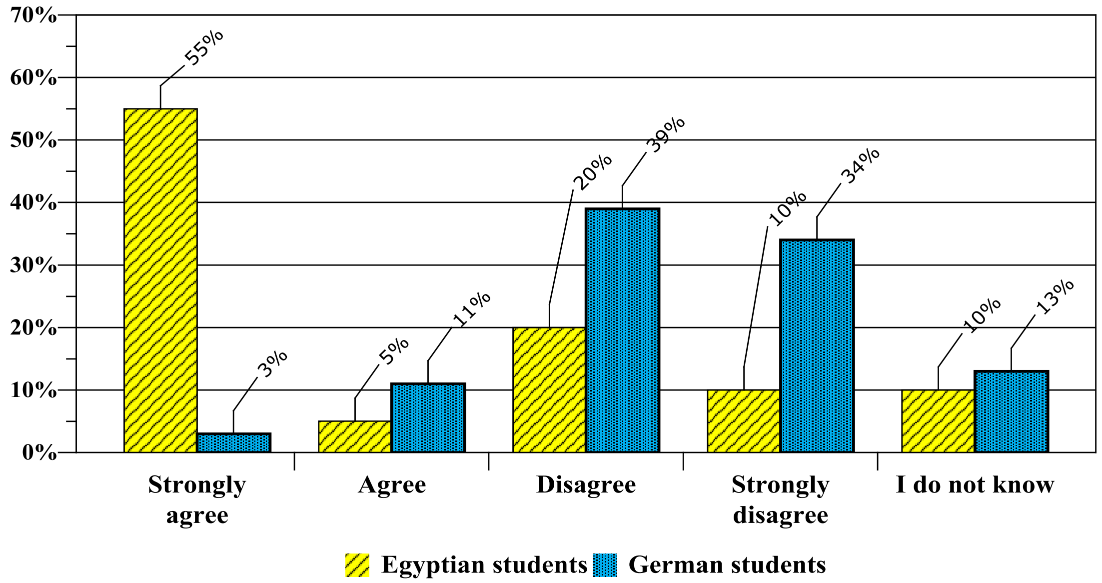


Figure 27: Difficulty of geometric proof

Item 7 was "*It is extremely hard to understand a geometric statement*." It was designed to examine to what extent the geometric statements are understandable in a traditional geometry class environment. In this item, the discrepancy between Egyptian and German students' responses was expected, while the traditional teaching method in Egyptian learning environment still has a meaning of one-way teaching; on the other hand it takes a different form to some extent in Germany, in which discussion and hands-on activities take place. Also, in the following table, the mean of Egyptian group responses is "2" which was equal to the response "Agree" and for German students it is "3.22" which fell approximately in the response "Disagree"

Table 5: Mean rank of item seven "Difficulty of understanding geometric statements"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
It is extremely hard to understand a geometric statement.	2.0000	1.07606	3.2222	0.76012

The figure gives a significant view of the above-mentioned discrepancy. The two most common responses for German students were "Disagree" and "Strongly disagree" accumulated in the positive side, while the Egyptian students' responses, recorded at 60%, were distributed between "Strongly agree" and "Agree", e.g., accumulated in the negative side. There are also more than 10% of German students who found difficulties in understanding geometric statements.

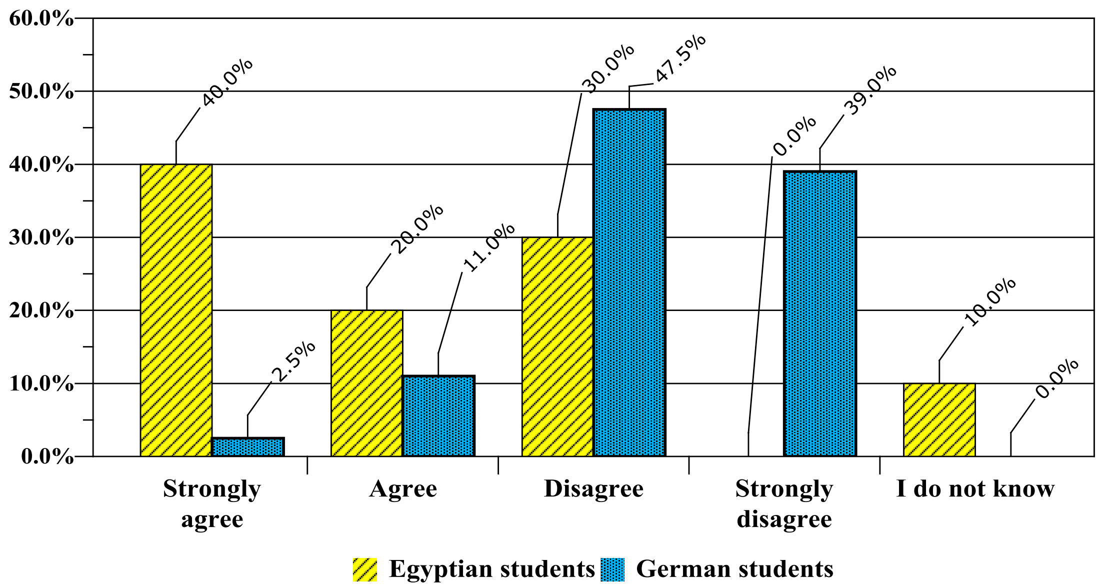


Figure 28: Difficulty of understanding geometric statements

Item nine in the questionnaire that students responded to was "*I can formulate a conjecture participating in discussions with teacher and class mates*." The following table shows that German students' responses fell mainly in "Agree", while the Egyptian students' responses tend to be "Disagree". That conforms to the previous result and reflects to some extent the discrepancy between the nature of the teaching methods in both Egypt and Germany.

Table 6: Mean ranks of item nine "Formulating conjectures through discussions"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
I can formulate a conjecture participating in discussions with teacher and class mates.	3.3500	0.74516	2.4722	1.05522

The figure below obviously shows those Egyptian students had difficulties in formulating conjectures and that they had never had experience of establishing these conjectures through discussions, neither with a teacher nor class partner. While more than 65% of German students' responses fell between "Agree" and "Strongly agree", the Egyptian students responded mainly between "Strongly disagree" and "Disagree". On the other hand, the figure shows an interesting result that there were also German students who disagree and strongly disagree with the item "I can formulate a conjecture participating in discussions with teacher and class mates."

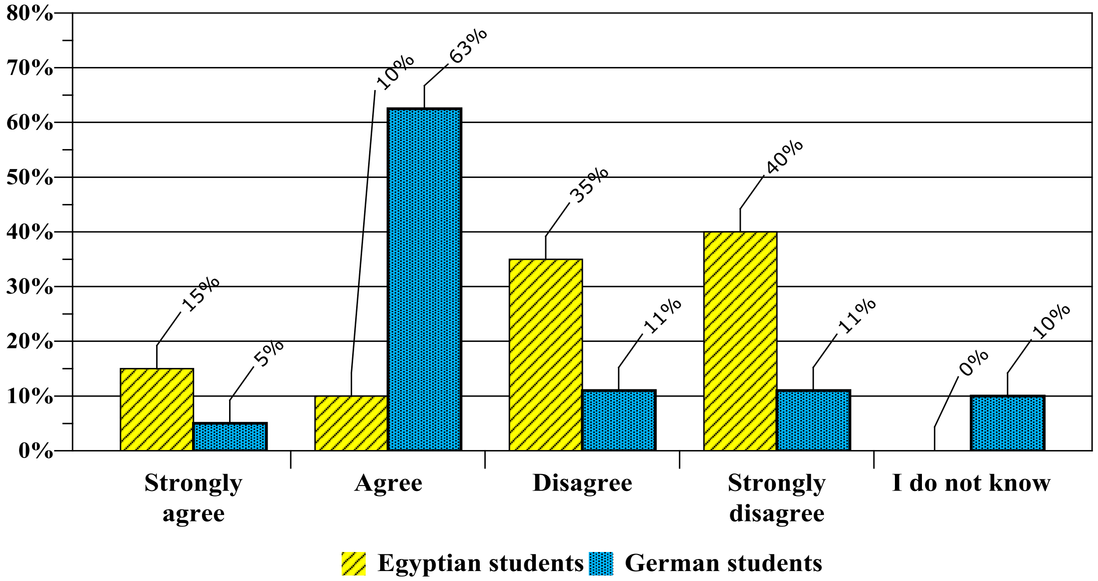


Figure 29: Formulating conjectures through discussions

Item ten in the questionnaire was designed to examine students' view of their ability to demonstrate their understanding of geometric proof. The result of students' responses was significant. The mean of Egyptian students' responses shows an agreement with the content of the statement, while the mean of German students' responses tended to reject it and tended to lie mainly under "Disagree".

Table 7: Mean ranks of item ten "Showing understanding of geometric proof"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
It is extremely hard to show my understanding of geometric proof.	1.8500	1.22582	2.6667	0.71714

The figure below gives the same conclusion, but it also provides interesting details about a cluster of German students – more than 20% – who agree on the difficulty of showing understanding of geometric proof, and approximately the same percentage of Egyptian students who do not find that it is extremely hard for them to show their understanding of geometric proof. Such results show the individual differences that might exist in both Egyptian and German learning environments which should be considered by both teachers and curricula developers.

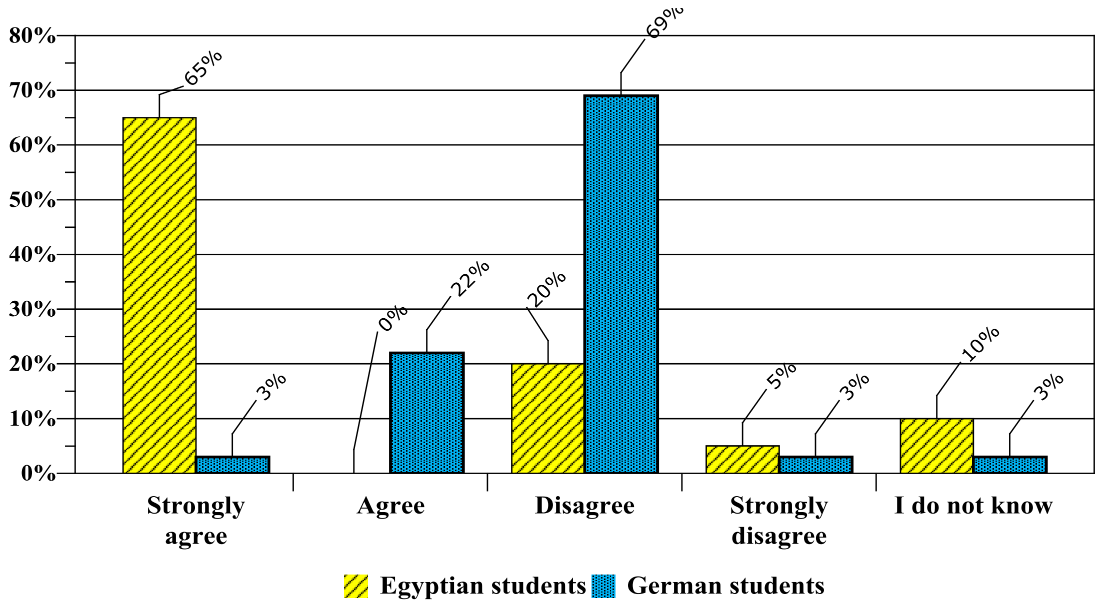


Figure 30: Showing understanding of geometric proof

The following table shows the mean of students' responses to item twelve that was designed to obtain insight about students' views of the explanation function of geometric proof. The mean of Egyptian students' responses shows a common disagreement, as shown in Figure 31. Most of their responses were 45% distributed between "Disagree" and "Strongly disagree". Also, an interesting observation is that 35% of Egyptian students and about 6% of German students responded with the choice "I do not know". When the seven students, who had given this response, were asked why they were not able to decide, most expressed their view about proof as being a tool of knowing and were convinced by the trueness of a geometric statement. Furthermore, through the discussion it was obvious that they had had no previous experience with such a task of explaining "why" a theorem is true. On the other hand, the mean of German students' responses tended to fall into responses ranging between "Agree" and "disagree" and the most common response was "Agree"; however, the figure shows that about 30% of German students responded between "Disagree" and "Strongly disagree".

Table 8: Mean ranks of item twelve "Realizing the explanation function of proof"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
Geometric proof gives me deeper understanding why the geometric statements are generally true.	1.8000	1.43637	2.6111	1.02198

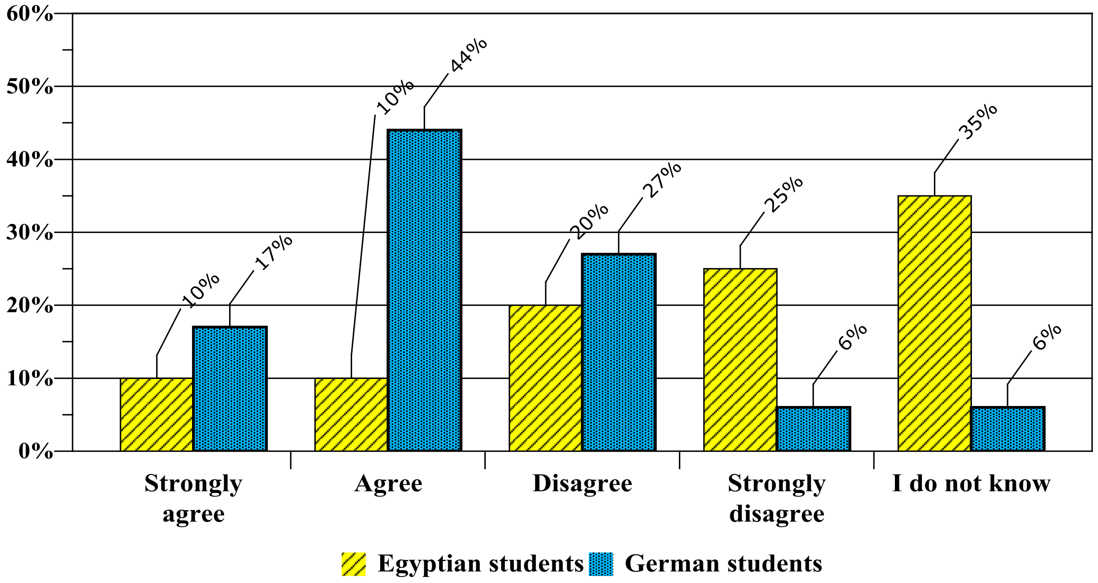


Figure 31: Realizing the explanation function of proof

With the same aim as the previous item, the following table shows the means of students' responses to item thirteen, which was also aimed to examine students' beliefs about further functions of geometric proof beyond verification. The mean of Egyptian students' responses fell between the values "1" and "2" which means between "Strongly agree" and "Agree", and from the figure we can conclude that there is disagreement in this item among about 50% of Egyptian students. It is also an interesting result, as was the case with the previous item, in that 40% of students had responded with "I do not know", while the mean of German students' responses shows to some extent the same result. It is also obvious that the percentage of the German students, 50%, who rejected the item, is more than the percentage of Egyptian students, 20%, who rejected the item.

Table 9: Mean ranks of item thirteen "Realizing other functions of proof"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
Often, I wonder: "why to proving on a statement, which is obviously true and proved by famous geometers!"	1.3500	1.38697	2.4444	0.96937

The result of this item emphasized what was concluded from item five, as both items give a view of German students' holding better beliefs in regard to geometric proof than those held by Egyptian students.

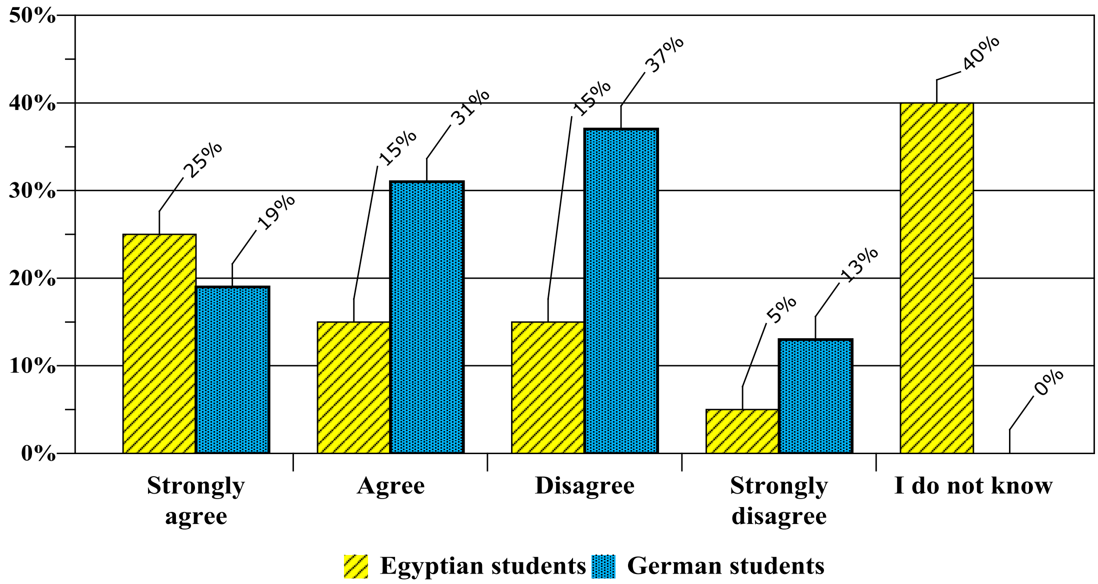


Figure 32: Realizing other functions of proof

The results of both Egyptian and German students' responses to item fourteen shows their false beliefs about the importance of proof in geometry, as most of the students' responses fell between "Strongly agree" and "Agree" with the percentage of agreement 50% of Egyptian students and about 68% of German students. While 45% of Egyptian students and about 30% of German students showed their disagreement on this item.

Table 10: Mean ranks of item fourteen "Feeling the importance of proof"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
I think, It is more important to use geometry statements to solve exercises more than to prove it.	3.2000	0.76777	2.0000	0.95618

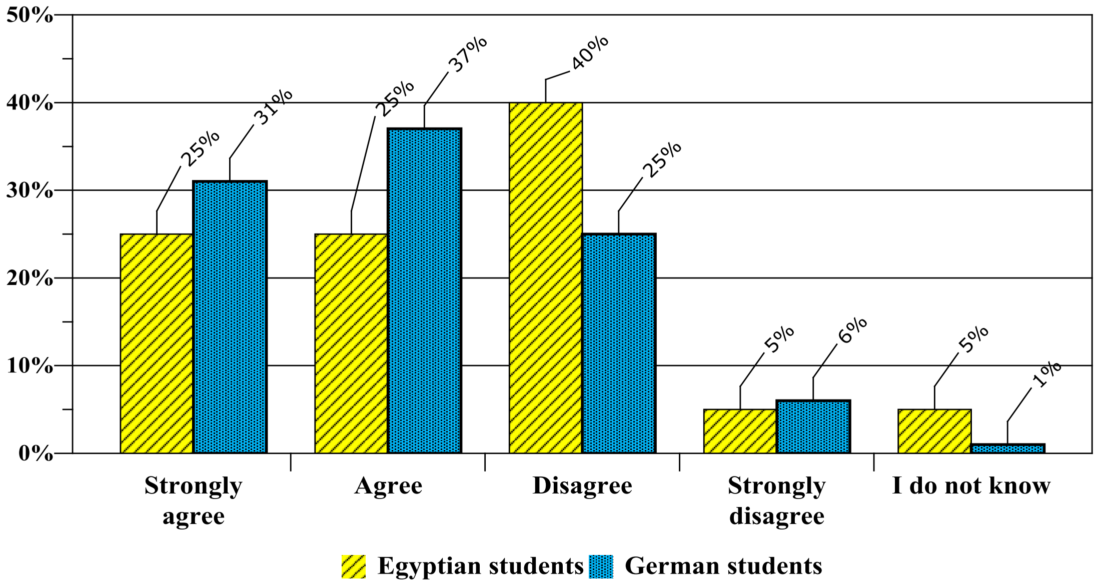


Figure 33: Feeling the importance of proof

Also, the result that could be concluded from students' responses to items 12, 13 and 14 is that there was widespread agreement among both Egyptian and German students that using the geometric statement in solving further exercises is more important than proving. In addition, the results also show the limitation of students' backgrounds regarding the functions of geometric proof to merely verify the rightness of geometric theorems. But the positive side that might be concluded is the better German students' emotions and beliefs about geometric proof than those held by Egyptian students.

The following item was included in the questionnaire with the aim of gathering information about the learning environment inside the geometry class, as to whether it enables students to collaborate and share their thinking. Table 10 shows that the mean of Egyptian students' responses fell mostly between "Strongly disagree" and "Disagree" in addition to 25% under the choice "I do not know" as shown in the figure below. In contrast, the German students commonly agree that they can work together collaboratively to complete the proof. This conclusion is consistent with what was previously concluded from students' responses to item nine, which examined the possibility of students' participation in discussions with the teacher and the class partner to formulate conjectures by themselves. These two conclusions emphasize the discrepancy between the nature of learning environment and teaching methods in both countries.

Table 11: Mean ranks item seventeen "Completing geometric proof through collaboration"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
I can complete a geometric proof through collaboration with my class mate.	1.1500	1.08942	2.6944	1.09073

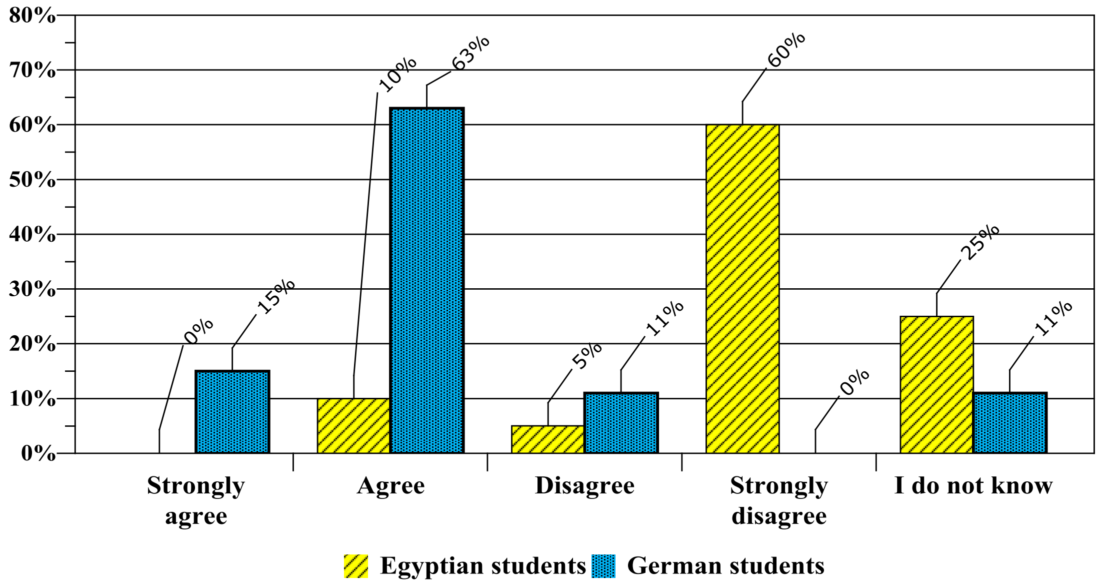


Figure 34: Completing geometric proof through collaboration

Item eighteen is one of the most significant items. Both Egyptian and German students seemed to respond around the same percentage of responses; 60% of Egyptian students agreed that concluding different observations in comparison to classmates and the teacher's conclusion led to feelings of frustration, the same result seemed to be applied to German students' responses.

This result sheds light on the importance of the teacher's role in geometry class to unify students' conclusions and their formulations, starting from their own language and transferring smoothly to the appropriate mathematical language.

On the other hand, the same percentage of disagreement, about 35%, is obviously apparent among Egyptian and German students as shown in the figure below.

Table 12: Mean ranks of item eighteen "Furstration feeling due to divergence in observations and conclusions"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
I feel frustrating when my observations are different from the conclusions of the teacher and other class mates.	2.6500	1.18210	2.2778	1.00317

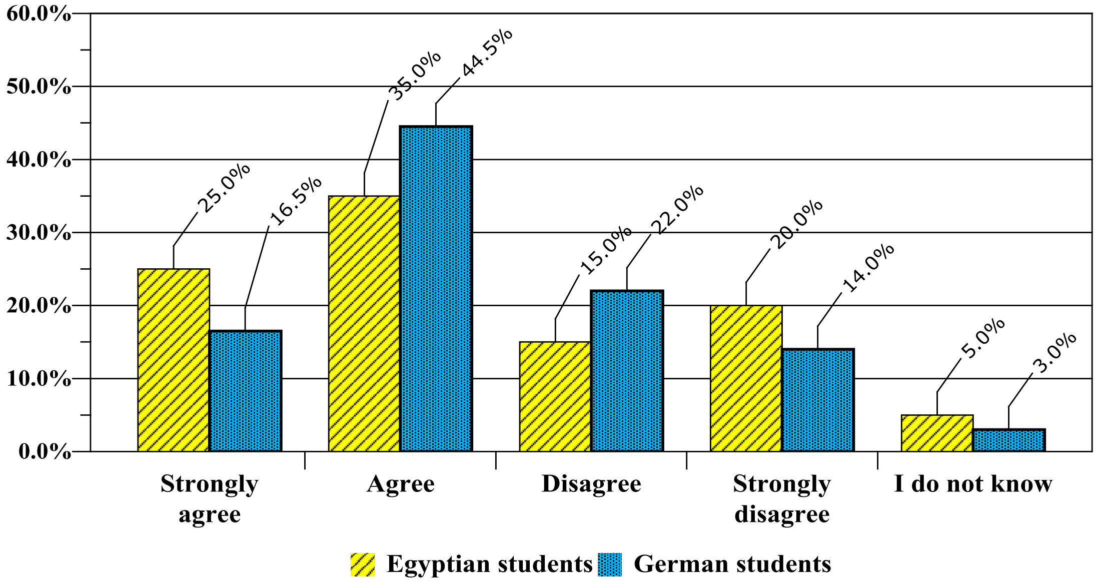


Figure 35: Furstration feeling due to divergence in observations and conclusions

4.1.2 Students’ responses to the open question about their view of geometry and geometric proof

The responses from the Egyptian students to the open question in the questionnaire of attitudes towards geometry and geometric proof show their profile that reflects the traditional teaching methods and the limited function of geometric proof in geometry class. One of the most significant responses came from one Egyptian student who stated “*I found proofs of theorems to be just steps to be memorized and the teacher prepares students to recall them in the exam and not for benefit of the theorems, and also we memorize the geometric theorems without really understanding the text*”

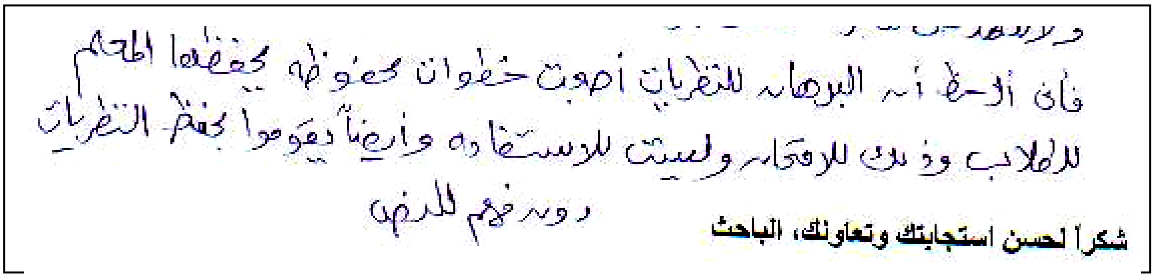

فإن الحظ أن البرهان للنظريات أصبح خطوات تحفظ يحفظها المعلم للطلاب وذلك للامتحان وليست للاستفادة وأيضاً يقوموا بحفظ النظريات دون فهم للنص
شكراً لحسن استجابتك وتعاونك، الباحث

Figure 36: An undergraduate student’s response to the open question

Another student’s response shows that despite his liking for mathematics, he feels fear and anxiety when faced with proving tasks. The student also expressed the difficulties in learning geometric proof as it includes different ideas for every problem which also requires a long time to solve.

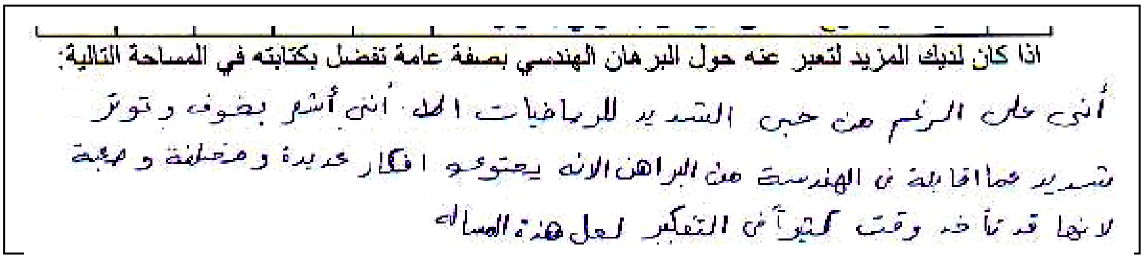

اذا كان لديك المزيد لتعبر عنه حول البرهان الهندسي بصفة عامة تفضل بكتابته في المساحة التالية:
أنني على الرغم من حبي الشديد للرياضيات الا أنني أشعر بخوف وتوتر شديد عندما اقابله في الهندسة من البراهين لأنه يحتوي افكار جديدة ومختلفة وصعبة لأنها قد تأخذ وقتاً كثيراً في التفكير لحل هذه المسأله

Figure 37: An undergraduate student’s response to the open question

4.2 Students' background and attitudes toward the computer use in mathematics and particularly in geometry learning

This part answers the second research question "*What are students' background and attitudes toward using computers in mathematics and particularly in geometry learning?*" according to students' responses "in both the Egyptian group and the German group before experiments" on 15 Likert-type items of the questionnaire of attitudes towards using computers in learning, in addition to their responses to an open question that enabled them to give further comments. In the following there is a review of the most significant students' responses before learning with the suggested dynamic geometry activities on the computer. This part of the result might give a view of students' emotions and beliefs about using computers in learning, while the second part summarizes students' comments in the open question about their learning experiences with learning software and particularly with dynamic geometry software.

4.2.1 Profile of students' attitudes toward the computer use

Item four in the questionnaire of attitudes toward using computers in learning was "*I hesitate to use a computer for fear of making mistakes*." The mean ranks of Egyptian and German groups fell between the two values "3" and "4" which means a negative item between "Strongly disagree" and "Disagree". This shows students' liking of and readiness to use computer without feeling anxious. This result encourages us to depend on the use of learning software as a means to motivate students to learn in addition to the expected efficiency of using the suggested dynamic geometry software to improve their attitudes towards geometry and geometric proof.

Table 13: Mean ranks of item four "Fear of using computer"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
I hesitate to use a computer for fear of making mistakes.	3.0500	1.27630	3.4444	1.02663

The following figure shows to some extent the similarity in the distributions of both Egyptian and German students' responses in the mentioned questionnaire item.

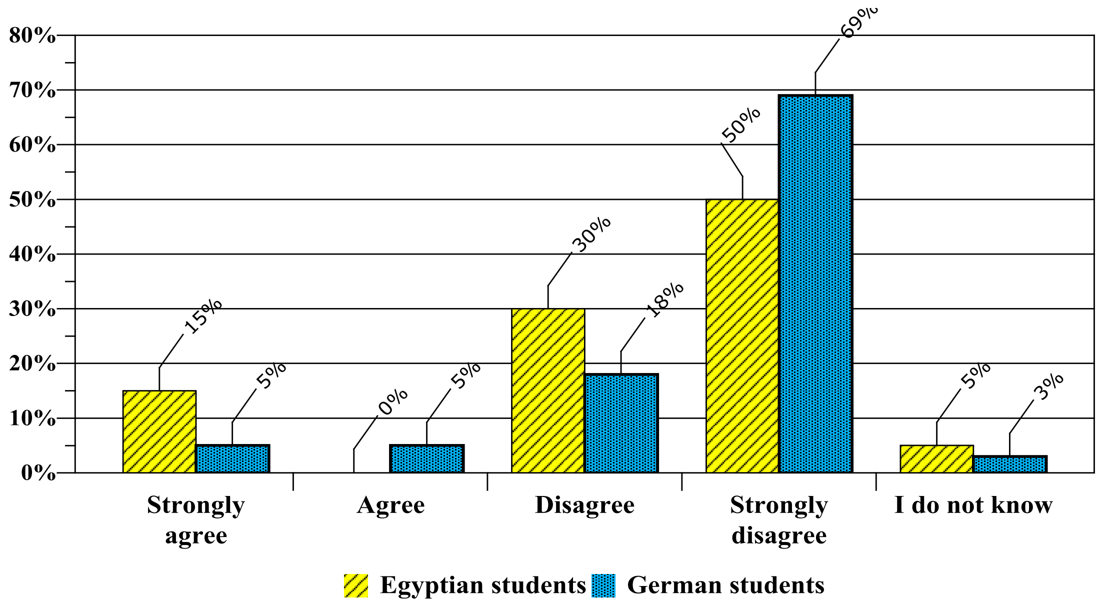


Figure 38: Fear from using computer

The next item supports what was concluded from the Egyptian students' responses in the above item. However, the responses of German students appeared to be contradicted, as about 70% of them disagreed that computers can motivate them to learn.

Table 14: Mean ranks of item eight "Computer and learning motivation"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
Computer can motivate me to learn.	2.9000	0.85224	1.9444	0.89265

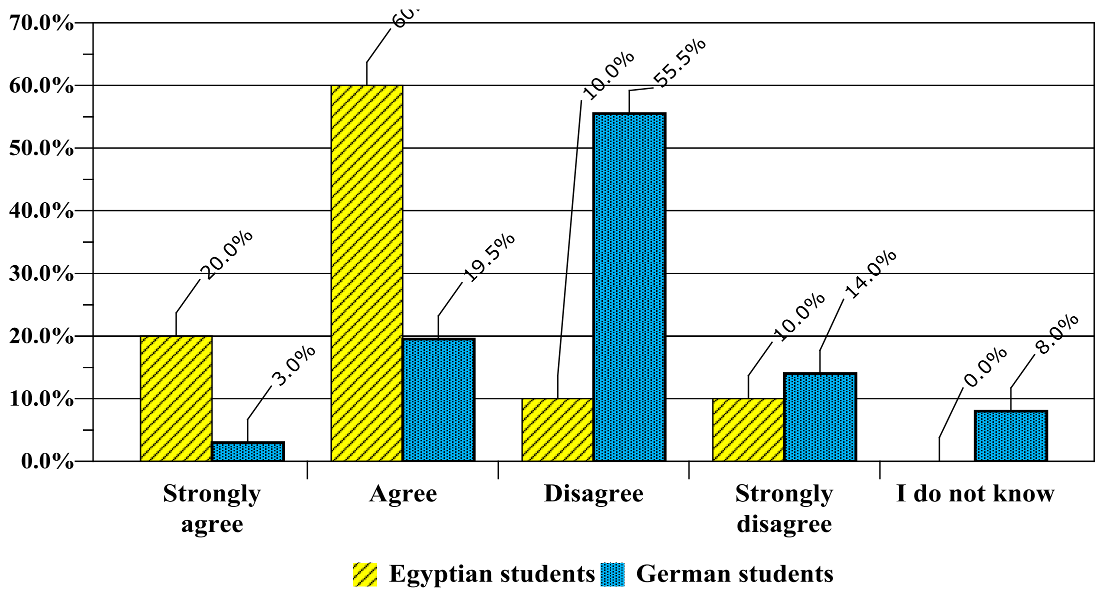


Figure 39: Computer and learning motivation

The following item was designed and included in the questionnaire in order to learn students' background with computers in facilitating the abstracted content. As the first aspect of any learning software was expected to be the visual illustration of information, the students' responses to that item shows an agreement among both Egyptian and German students that computers can help them to understand and visualize the abstracted information, while the main ranks of both two groups tended to be "3" and that means the students' responses as a whole fell in "Agree". The figure also shows that both groups have approximately the same percentage of agreement at around 80%; on the other hand, around 15% of students' responses fell in the Disagreement zone.

Table 15: Mean ranks of item nine "Facilitating abstract content"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
Computer helps me to understand the abstract content throughout visualization.	3.0000	1.25656	2.9444	0.79082

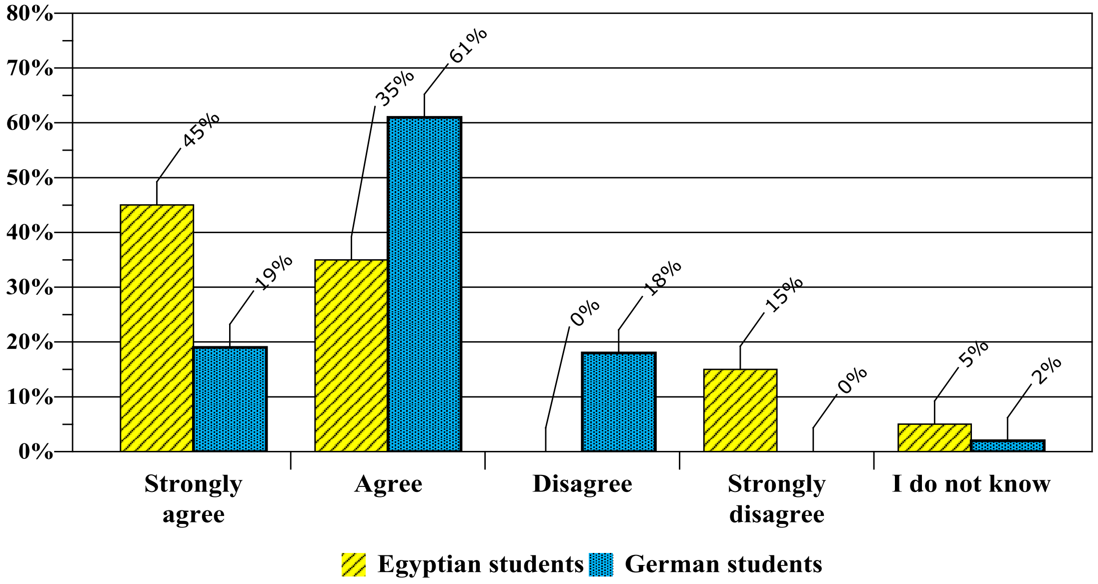


Figure 40: Facilitating abstract content

This result suggests that the efficiency of dynamic geometry software in benefits from students' visual ability to stimulate their observation and then provide an explanation. This aspect might facilitate the smooth transfer from just the visual recognition in the beginning through students' explanation to the level of proof and justification.

The next item about the role of computers in stimulating students' interest shows that Egyptian students' responses were mostly around 70% and fell between "Strongly agree" and "Agree", which explains the needs for a learning environment that increases interest and stimulates their motivation towards learning. The result of Egyptian students' responses to this item seemed to be consistent to some extent with the previous item, which showed a possible increasing in their motivation through using computers in learning. However, like the contradiction in German students' responses to the previous item about learning motivation, about 60% of their responses fell in the disagreement zone.

Table 16: Mean ranks of item eleven "Stimulating students' interests"

	Egyptian students		German students	
	Mean	St. Dev.	Mean	St. Dev.
Use the computer in teaching adds interest to learning.	2.6500	1.08942	2.1389	0.99003

The figure also shows that the mentioned responses of German students were distributed on a normal curve, while Egyptian students' responses tailed positively in the agreement area.

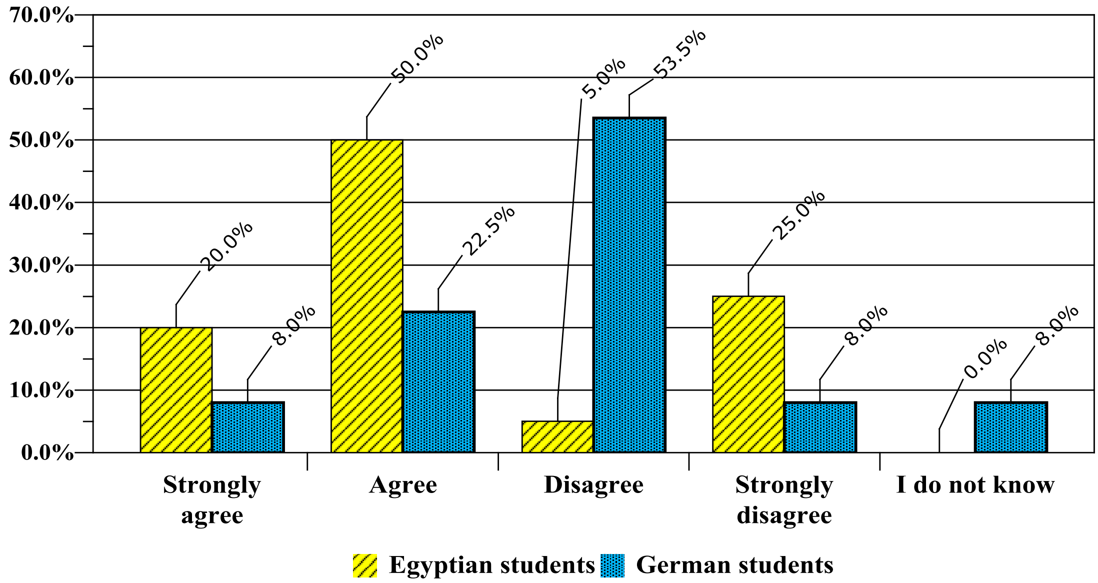


Figure 41: Stimulating students' interests

This result invited researchers in Egypt to investigate the nature of learning environments in other countries such as Germany, in order to enable students to be more interrested toward learning topics, especially in abstracted fields such as mathematics.

4.2.2 Students' background about the computer use in mathematics

The students' responses to the open question "*Did you have an experience with any mathematics learning software?*" might give insight into their background about using computers in mathematics learning in addition to their background about mathematical learning software and particularly dynamic geometry software.

The students in this question have to choose between "Yes" they have a previous experience or "No" if they have never had this learning experience with any mathematical software. If the responder chooses yes, he is asked about the name of that software he has learned with, and which advantages this software has?

The Egyptian students' responses showed that they have no previous experience with dynamic geometry software as most of them choose "No" and the students who responded with "Yes" mentioned what could be so-called "one-way presentation software" that depends on only multimedia presentations using texts, sound and videos.

In comparison, German students seemed to a wide extent to have had more experience using computers in learning especially in mathematics learning, and in particular with dynamic geometry software.

The following table shows the numbers and percentages of students' responses distribution that presents their previous experiences with mathematical software and especially with the different dynamic geometry software they mentioned. Only 72 students from 200 German students have previous experience with mathematical software.

Table 17: Mathematical software that the German students have mentioned

Name of the mathematical software	**Forgotten**	**Excel**	**SPSS**	**Mathbits (website)**	**Auto CAD**	**Derive**	**Maple**	**Mathematica**	**Euklid**	**Dynageo**	**Cinderella**	**GeoGebra**	**GEONExT**	**Total**
The number of students	20	3	1	1	1	3	9	1	2	1	15	6	9	**72**

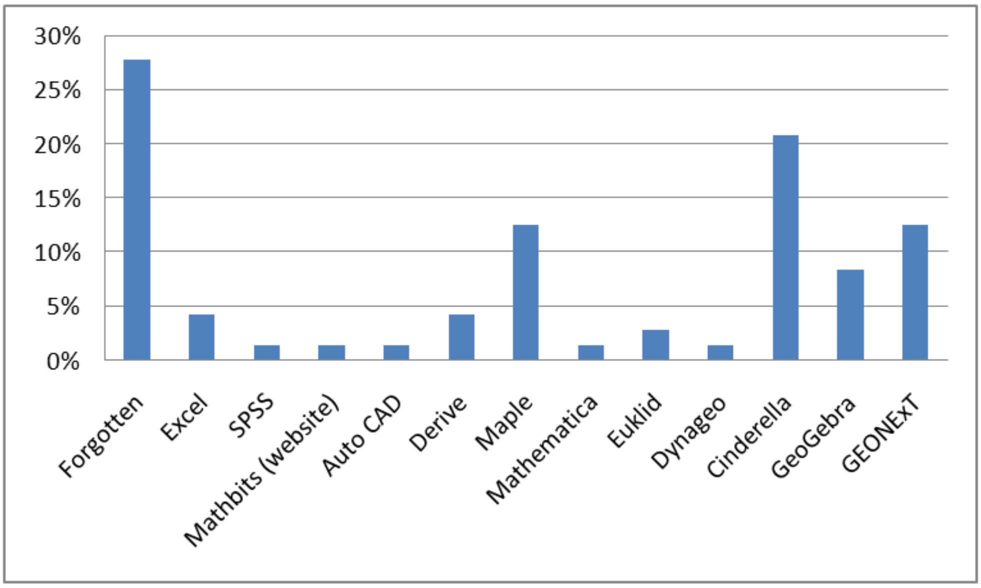


Figure 42: The distribution of mathematical software mentioned by German students

The above figure shows that about 27% of the German students who responded with "Yes" I have previous experience cannot remember the name of the software they have used. On the other hand, about 10% of students mentioned software such as Excel, SPSS, Auto CAD and Mathbits website, which is not considered as dynamic geometry software. Also, the thing that surprised the present researcher was the number of German students who really have had previous learning experience with dynamic geometry software. Out of 200 German students only 13 students had used one of the dynamic geometry software packages in their previous school time or at university. Also, the common features that the thirteen students mentioned in their responses to the open question in the questionnaire, were the ease of constructing and learning using this software compared to sketching on paper, the possibility to illustrate and visualize difficult and abstracted mathematical and geometrical problems, and one student stated the possibility of posing and extending problems to reach a deeper understanding.

The students also agreed on one advantage of avoiding calculation errors and the facilities of measuring tools to get the basic ideas required to solve problems. In addition, one student mentioned that we can construct, for example, curves and ideas come out further through discussions.

4.3 The effectiveness of the suggested approach

The Wilcoxon rank test and the effect size coefficient were used in analyzing the ranked data collected through the three questionnaires, as an equivalent of the paired t-test in addition to the descriptive statistical analysis of the three questionnaire items. The statistical package for social sciences 'SPSS' was used for inputting and processing the data collected from the questionnaires before and after intervention. Thus, this part answers the third research question, which was concerned with improving students' attitudes towards geometry and geometric proof.

Table 18: Wilcoxon Ranks Test of attitudes towards geometry

Ranks		N	Mean Rank	Sum of Ranks
After-Before	Negative Ranks	4[a]	2.50	10.00
	Positive Ranks	5[b]	7.00	35.00
	Ties	3[c]		
	Total	12		

a: After < Before, b: After > Before and c: After = Before

Table 19: Wilcoxon Ranks Z Test statistics

	After- Before
Z	-1.482[a]
Sig. (2-tailed)	0.138

a: based on positive ranks and b: Wilcoxon Ranks Test

Based on the coefficients produced from the above tables, the results indicated that there are general differences in students' attitudes towards geometry and geometric proof, $z = -1.482$, but these differences were not significant because the significant coefficient Sig. = 0.138 is higher than the confidence interval $\alpha = 0.05$; this result showed that individual item analysis is needed in order to investigate these differences since significant differences in individual item responses may exist.

However, while the previous result does not show an overall significant effectiveness for the suggested approach on improving attitudes, this does not finally mean a denial of possible significant gains in some of the students' views of geometry and geometric proof. The students' responses to individual questionnaire items might indicate different conclusions.

This assumption might proceed from these results of Almeida's (2000) questionnaire. Despite the general positive attitudes towards geometry, the results of Almeida's questionnaire indicated that students have false beliefs about the functions of geometry proof which are limited to the verification function.

This shows the need for further analysis and investigation. It also reflects the need for longer intervention in order to improve students' attitudes towards geometry and geometric proof. The following shows further analysis regarding the most significant gains between administering the questionnaire of attitudes towards geometry and geometric proof before intervention, and administering the questionnaire about the suggested approach after intervention.

The same statistical method using the Wilcoxon rank test, with the accepted confidence interval $\alpha = 0.05$.

The first comparison is between students' responses to item seven "*It is extremely hard to understand a geometric statement*." in the questionnaire of attitudes towards geometry and geometric proof and item eleven "*It is easier to formulate and understand a geometric statement using dynamic geometry activities*." in the questionnaire of the suggested approach.

Table 20: A comparison between mean ranks of item 7 “toward proof” before intervention and item 11 “toward the suggested approach” after intervention

	N	Mean ranks	Std. Deviation	Z	Sig. (2-tailed)
prpre7	20	2.00	1.07606	-1.43	0.037
DGS11	12	3.50	0.67420		

The significant coefficient Sig. = 0.037 is lower than the confidence interval α = 0.05; this result indicates that using dynamic geometry activities facilitates formulating and understanding of the included geometric statements.

The next comparison is between students’ responses to item eight “*I can get the key idea to start proving geometric statement.*” in the questionnaire of attitudes towards geometry and geometric proof, and item seven “*Dynamic geometry software enables me to get the idea of proof easier.*” in the questionnaire of the suggested approach.

Table 21: A comparison between mean ranks of item 8 “toward proof” before intervention and item 12 “toward the suggested approach” after intervention

	N	Mean ranks	Std. Deviation	Z	Sig. (2-tailed)
prpre8	20	2.70	1.26074	-1.23	0.041
DGS7	12	3.42	0.66856		

The significant coefficient Sig. = 0.041 is lower than the confidence interval α = 0.05, this result indicates that using dynamic geometry activities stimulates and facilitates getting the key idea of proof.

The third comparison between students’ responses to item seventeen “*I can complete a geometric proof through collaboration with my class mate.*” in the questionnaire of attitudes towards geometry and geometric proof, and item thirteen “*Dynamic geometry activities enable me to collaborate with my colleagues, which facilitate geometry proof learning.*” in the questionnaire of the suggested approach.

Table 22: A comparison between mean ranks of item 17 “toward proof” before intervention and item 13 “toward the suggested approach” after intervention

	N	Mean ranks	Std. Deviation	Z	Sig. (2-tailed)
prpre17	20	1.15	1.08942	-2.23	0.013
DGS13	12	3.08	0.90034		

The significant coefficient Sig. = 0.013 is lower than the confidence interval α = 0.05; this result suggests that the dynamic geometry activities facilitates geometry learning through collaborating and sharing ideas between class partners.

In Table 23 the comparison is between students’ responses on item nineteen “*In geometry class, the teacher presents proofs with rarely use of discussions.*” in the questionnaire of attitudes towards geometry and geometric proof, and item seventeen “*I found it is useful to get support from the teacher during learning with dynamic geometry activities.*” in the questionnaire of the suggested approach.

Table 23: A comparison between mean ranks of item 19 “toward proof” before intervention and item 17 “toward the suggested approach” after intervention

	N	Mean ranks	Std. Deviation	Z	Sig. (2-tailed)
prpre19	20	1.65	1.18210	-2.13	0.033
DGS17	12	3.08	1.37895		

The significant coefficient Sig. = 0.033 is lower than the confidence interval α = 0.05; this result reflects that using the suggested activities opened up a possible communication channel between students and the teacher, while the students in a one-way teaching environment miss the proper help for achieving the targeted learning goals. With consideration that the Egyptian students completed only six activities from the entire suggested treatment content, the above-presented results show two various benefits of the suggested content: Firstly, the improvement in attitudes towards geometry and geometric proof and, secondly, overcoming some of the students’ learning difficulties, particularly in understanding geometric statements and approaching the level of formal proof, in addition to changing to some extent the traditional frontal teaching method and establishing a collaborative geometry-learning environment that also open the channels of discussions among students and between students and the teacher.

In comparison to their views on using computers in learning, the following section will present students' attitudes towards the suggested approach and its content.

4.4 The students' attitudes towards the suggested approach

The gathered data from the questionnaire of attitudes towards the suggested approach was ranked data, which also required nonparametric statistical processing. Thus in the following tables, the Wilcoxon rank test and the effect size coefficient were used, to examine the gain between students' experiences with using computers in learning and their new experience with using the suggested approach and the included dynamic geometry activities, through the study experiments.

Table 24: Differences between attitudes towards computer and the suggested approach

Ranks		N	Mean Rank	Sum of Ranks
Suggested Approach-Computer	Negative Ranks	1[a]	1.00	1.00
	Positive Ranks	11[b]	7.00	77.00
	Ties	0[c]		
	Total	12		

a: After < Before, b: After > Before and c: After = Before

Table 25: Wilcoxon Ranks Z Test statistics

	Suggested Approach – Computer
Z	-2.981[a]
Sig. (2-tailed)	0.003

a: based on positive ranks and b: Wilcoxon Ranks Test

The coefficients in the previous tables show a significant difference between students' attitudes towards using a computer in learning and towards the suggested approach which they intended, while the coefficient Sig. = 0.003 is significantly higher than the value of α = 0.05 "e.g., as the confidence interval = 0.95" in most of the previous studies and even if compared with the value of α = 0.01. "With the confidence interval 0.99". This result invites the researcher to use the following equation in order to compute the effect size: $r = Z/\sqrt{N}$, N is the number of students and Z is Z-score of Wilcoxon Ranks Test.

Thus $r = -2.981/\sqrt{12} = -2.981/3.46 = -0.86$

The coefficient "r" according to the table of critical values for the Wilcoxon test indicates that this difference is substantial and shows positive attitudes towards using the suggested approach which contains the real-life geometric situations which were based on dynamic geometry software. The following table gives an overview of students' responses from the suggested approach questionnaire, and shows the mean ranks of Egyptian students' responses and their equivalent meanings.

Table 26: The mean ranks of responses on the questionnaire of suggested approach

N	Statement	Mean ranks	Equivalent response
1	Using dynamic software is a new experience for me.	4	Strongly agree
2	I enjoyed learning with dynamic geometry activities.	3.67	Agree – Strongly agree
3	I have learned a lot using dynamic geometry software.	3.5	Agree – Strongly agree
4	It is useful to learn proof using dynamic geometry activities.	3.25	Agree
5	Dynamic geometry activities facilitated geometry proof learning.	3.33	Agree
6	My interest to proof learning increased using dynamic geometry activities.	3.5	Agree – Strongly agree

N	Statement	Mean ranks	Equivalent response
7	Dynamic geometry software enables me to get the idea of proof easier.	3.42	Agree
8	Using dynamic geometry activities facilitate completing proof step by step.	3.58	Agree – Strongly agree
9	It is useful to find help when needed to complete proving.	3.42	Agree
10	Dynamic geometry activities facilitate conjecturing throughout experimenting.	3.42	Agree
11	It is easier to formulate and understand a geometry statement using dynamic geometry activities.	3.5	Agree – Strongly agree
12	I found the daily-life dynamic geometry activities are useful.	3.58	Agree – Strongly agree
13	Dynamic geometry activities enable me to collaborate with my colleagues, which facilitate geometry proof learning.	3.08	Agree
14	Dynamic geometry activities are not only one time presentations but also give different possibilities to understand.	2.83	Agree
15	Dynamic geometry activities allow me to learn in my own learning tempo.	3.2	Agree
16	I have learned better than the teacher can do without this new approach.	3.5	Agree – Strongly agree
17	I found it is useful to get support from the teacher during learning with dynamic geometry activities.	3.08	Agree
18	Dynamic geometry activities make geometry class boring.	3.33	(Negative statement) Disagree
19	Dynamic geometry activities make geometry class enjoyable.	3.42	Agree

N	Statement	Mean ranks	Equivalent response
20	Teacher allows me to recognize the properties of geometry construction using dynamic geometry software better than stable drawing on the blackboard.	3.25	Agree

Consistent with the result of the Wilcoxon test, the above table obviously shows that all of the students' responses fell in the Agreement range regarding the efficiency of the suggested approach and its content in geometry and geometric proof learning.

Furthermore, the most notable of the students' responses to the open question about the suggested approach and in particular to using dynamic geometry software, came from two students. The first student stated that "*learning proof in the dynamic geometry environment is better than the traditional method because it enables several sequenced chances of gaining a deeper understanding in an easy way and at my own tempo*"

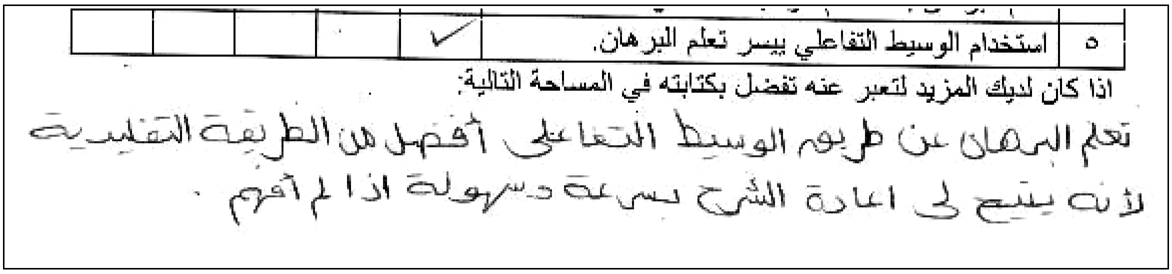


Figure 43: Response of Student 'A' to the open question about the suggested approach

The second student stated that "*I can understand proofs in dynamic geometry learning activities better than the teacher's presentations on the board, while dynamic geometry facilitates approaching the proof in such way that I will never forget*"

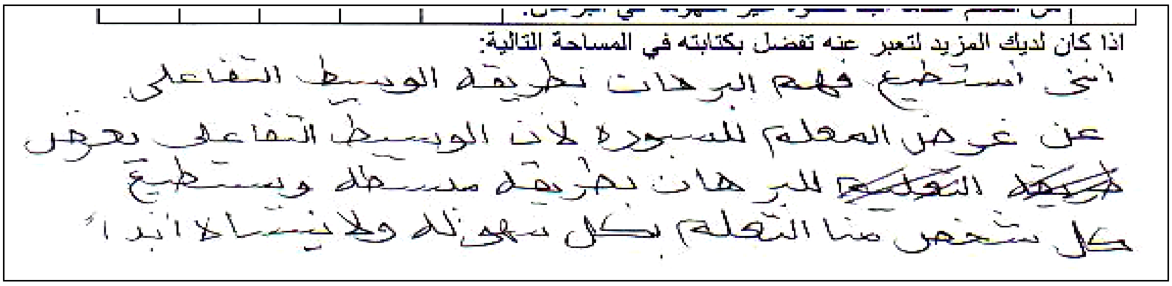


Figure 44: Response of Student 'B' to the open question about the suggested approach

Also, through the sessions of the main study experiments and students' worksheets, it was observable that the Egyptian students were able to finish the real-life activities earlier than the group of German students, as the Egyptian students afterwards studied two more abstracted geometric activities related to "Wallace line". Thus, what limits the results of the present study to the group of Egyptian students is the drop-out rate in the group of German students. Only six students subjected themselves to the study experiments out of about 200 German participants in the pretest phase. Unfortunately, only one student was available for the post test. It was also obvious that the six German participants needed far more time not only to finish every real-life geometry task, but also at the beginning when familiarizing themselves with how to work through the geometric activities. Furthermore, and according to the entrance test, a large number of the German students, who attended the pretest phase, do not have the geometric prerequisites to study the content of the activities, for instance, some of them were confused in distinguishing between similarity and congruency of triangles, which is supposed to have been mastered before entering the level of university education.

However, the results obtained from the 200 students who participated in the pretest phase were very important for showing the discrepancies between them and the group of Egyptian students regarding their backgrounds in using computers and mathematical learning software, their views about the importance of geometry and their beliefs about the functions of geometric proof which obviously shows that there are real learning difficulties and false beliefs among Egyptian students, especially those who will be a mathematics teachers in the near future.

5 Conclusion

The basic argument in the brainstorming phase of the present study was why the geometers, who discovered numerous geometric concepts and theorems, were motivated to explain and prove the correctness of their conclusions and, secondly, supposing that we invited a famous geometer into our traditional geometry class to study and prove a new theorem that he did not know, do you think that the geometer would be motivated to learn and prove the correctness of the theorem directly without participating in exploring it and the included relations?

If the answer is no, or at best ‘maybe’, how can we expect a positive attitude and high achievement from our students in such a learning environment, if their roles are merely to receive knowledge in a one-way channel of communication. A consequential and critical phenomenon in Egyptian schools and even universities is that a wide range of students are opting against specializing in mathematics.

The present researcher believes that the primary goal of educational research in a country such as Egypt, which still has many educational problems, should be to investigate the possibilities to overcome these problems in different learning subjects.

5.1 What was gained by conducting this study?

The present study disclosed the facilities of dynamic geometry software in utilizing learning theorems such as cognitive and social constructivism in developing an appropriate learning environment for overcoming several difficulties, false beliefs and negative attitudes in geometry class.

Also, the overall outcome of the present study is the presentation of a suggested approach that adds a new contextualized organization for geometric content, which could be termed “Story-based Dynamic Geometry”, which tries to enable students to be able to go through the phases of discovering geometric concepts and theorems and then reach the level of proving.

Moreover, it presents geometric concepts and theorems through real-life situations, which could enable students to feel the importance of geometric knowledge in the real world. Likewise, the present researcher was also concerned with the benefits to be gained from dynamic geometry software to engage students in a virtual hands-on experience for geometric concepts and theorems, and uncover the beautiful of geometry and enable them to rediscover geometric concepts and theorems.

Thus, the suggested approach in the present study was a means of improving attitudes towards geometry and geometric proof of undergraduate students in the Faculty of Education as they are aiming to be mathematics teachers in the near future. The importance of the selected undergraduate students as the subjects of the study is in the assumption that they will be the source of positive attitudes and role models for their future students.

5.2 Outside Factors

In the following and before truly significant results can be derived from this work, there are two factors which may have had an impact on the results and which must be considered:

1. The limited time available for students to participate in voluntary activities outside of their study for other courses.

2. The drop-out in the group of German students limits the results to the group of Egyptian students. However, the results of the pretest phase were very important for showing the discrepancies between them and the group of Egyptian students regarding their backgrounds in using dynamic geometry software and their beliefs about the functions of geometric proof.

5.3 Significant results

This research work tried to enable students to feel the importance of geometry in daily life, to share the process of formulating geometric statements and conjectures, to realize geometric proof as being more than a tool of validating the correctness of geometric theorems, as well as starting from a real-life situation and going through the reasoning processes that might evolve students' geometric thinking smoothly to reach the level of geometric proof. The results presented in the previous chapter showed that the suggested approach was effective in:

1. Enabling students to formulate and understand geometric statements (e.g., see the first comparison and Table 20 on pages 138, 139).

2. Stimulating students to experiment and facilitating then to get the key idea of proof (e.g., see the comparison and Table 21 page139).

3. Stimulating students to collaborate and share ideas (e.g., see the third comparison and Table 22, pages139, 140).

4. Enabling students to experiment and facilitating in them getting the key idea of proof (e.g., see the comparison and Table 23 on page 140).

5. Increasing students' interest in proof learning (e.g., item 6, page 142)

6. Facilitating completing proof step by step (e.g., the mean rank of students' responses on item 8, page 143).

7. Allowing the student to learn at his own learning tempo (e.g., item 15, page 143 and response of students A to the open question on page 144).

8. Increasing retention of proof learning and understanding (e.g., response of students B to the open question on page 144)

9. In addition, the results also indicated that the students' attitudes towards the suggested approach were very positive (e.g., see the results on pages 141-144).

The above shows the effectiveness of using the suggested approach in overcoming some geometry-learning difficulties and improving students' participations in geometry class, particularly when it includes proof tasks. In contrast, the analysis of the collected data does not indicate an overall significant difference in students' emotions towards geometric proof (e.g., see the results of Wilcoxon rank test on page 137). The limited duration of the study experiments might explain this result. As the participants might not have had enough time to learn using the suggested content.

5.4 Future considerations

Accordingly, the present researcher has some recommendations and suggestions for developers of mathematical curricula and for future studies, which are summarized in the following points:

1. Developers of mathematical curricula might benefit from the present study in organizing and designing mathematics content in a story context that relates school geometry to real-life applications, and might preserve students' motivations.

2. Mathematics teachers and developers, particularly in Egypt, could benefit from the facilities of dynamic geometry software in developing and improving mathematics-learning environments.

3. Future studies conducted over a longer period of time may produce more positive results.

4. Allow students to participate in structuring geometric situations and in constructing the dynamic geometry applets; while the Egyptian students who participated in the present study had no previous experience with using dynamic geometry in mathematics learning.

5. The present study can be replicated to investigate attitudes towards geometry and geometric proof in each level of geometric thinking.

Appendix A: Abbreviations

AutoCAD	Computer Aided Engineering Design Software
Cinderella	Dynamic Geometry Software
Derive	Mathematical and Analytical Software
DGS	Dynamic Geometry Software
Dynageo	Dynamic Geometry Software
Euklid	Dynamic Geometry Software
Excel	Spreadsheet Application distributed by Microsoft
GeoGebra	Dynamic Geometry Software
GEONExT	Dynamic Geometry Software
GSP	Geometer’s Sketchpad
Java	A programming language and computing platform released by Sun Microsystems
JavaScript	A Programming Language that makes Web pages Interactive
Maple	Mathematical and Analytical Software
Mathbits	Mathematics Learning Website
Mathematica	A computational software
NCTM	National Council for Teachers of Mathematics
SPSS	Statistical Package for Social Science
ZPD	Zone of Proximal Development

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Appendix C: Questionnaire of Attitudes towards Geometry and geometric Proof

Questionnaire of Attitudes towards Geometry and Geometric Proof

Dear students

This questionnaire is a part of educational research in geometry.

It aims at understanding your emotions and beliefs about proof learning in geometry class.

Any personal information obtained from this questionnaire will be confidential. Moreover, to feel more comfortable, write your name if you want or write a code that you may use again.

Please, read every item carefully before responding and when you feel hesitate, please put your sign under the column "I don't know"

Name or ID code: (please remember your ID)............................... **Gender**: ....

N	Statement	Strongly agree	Agree	Disagree	Strongly disagree	I don't know
1	I enjoy geometry class when it includes proving tasks.					
2	Geometry is my favorite topic in mathematics.					
3	Geometric proof is easier than other topics in mathematics.					
4	I feel upset and confuse when I am doing poorly in proof tasks.					
5	Proof is extremely hard so that it takes the enjoyment of learning away.					
6	I need much time to understand geometric proof more than other topics.					
7	It is extremely hard to understand a geometric statement.					

N	Statement	Strongly agree	Agree	Disagree	Strongly disagree	I don't know
8	I can get the key idea to start proving geometric statement.					
9	I can formulate a conjecture participating in discussions with teacher and class mates.					
10	It is extremely hard to show my understanding of geometric proof.					
11	Often, I cannot get help to complete my geometric proof.					
12	Geometric proof gives me deeper understanding why the geometric statements are generally true.					
13	Often, I wonder: "why to proving on a statement, which is obviously true and proved by famous geometers!"					
14	I think, It is more important to use geometry statements to solve exercises more than to prove it.					
15	I have learned how to formulate and generalize geometric statements.					
16	In any geometric task, it is clear for me what the givens are and what is required to prove.					
17	I can complete a geometric proof through collaboration with my class mate.					
18	I feel frustrating when my observations are different from the conclusions of the teacher and other class mates.					
19	In geometry class, the teacher presents proofs with rarely use of discussions.					

N	Statement	Strongly agree	Agree	Disagree	Strongly disagree	I don't know
20	I feel frustrating when I do not get a comment on my answers or participations in geometry class.					
Further Comments						

Thank you for your participating.

Appendix D: Questionnaire of attitudes towards using computer in learning

Questionnaire of attitudes towards using computer in learning

Dear students

This questionnaire is a part of educational research in geometry.

It aims at understanding your opinion about using computer in learning.

Any personal information obtained from this questionnaire will be confidential. Moreover, to feel more comfortable, write your name if you want or write a code that you may use again.

Please, read every item carefully before responding and when you feel hesitate, please put your sign under the column "I don't know"

Name or ID code: (please remember your ID).............................. **Gender**: ....

N	Statement	Strongly agree	Agree	Disagree	Strongly disagree	I don't know
1	I enjoy using the computer in learning.					
2	I need help to know how to use the computer.					
3	It is impossible for me to use the computer regularly in learning.					
4	I hesitate to use a computer for fear of making mistakes.					
5	Computer makes learning boring.					
6	I can learn better without the computer.					
7	Learning with a computer requires more time than traditional methods.					

N	Statement	Strongly agree	Agree	Disagree	Strongly disagree	I don't know
8	Computer can motivate me to learn.					
9	Computer helps me to understand the abstract content throughout visualization.					
10	I use my computer in learning at home.					
11	Use the computer in teaching adds interest to learning.					
12	The teacher uses computer to enhance the presentations.					
13	Computer makes it possible to learn better than the teacher can do with the blackboard.					
14	Computers make learning tasks related to life.					
15	Teaching using the computer gives the opportunity for discussions in the class.					

Did you have an experience with any mathematics learning software?

Yes ☐ No ☐

If yes, Which software?

Which advantages have this software?

Thank you for your participating.

Appendix E: Questionnaire of attitudes towards the suggested approach

Questionnaire of attitudes towards the suggested approach

Dear students

This questionnaire is a part of educational research in geometry.

It aims at understanding your opinion about the suggested learning activities.

Any personal information obtained from this questionnaire will be confidential. Moreover, to feel more comfortable, write your name if you want or write a code that you may use again.

Please, read every item carefully before responding and when you feel hesitate, please put your sign under the column “I don’t know”

Name or ID code: (please remember your ID).............................. **Gender**: ....

N	Statement	Strongly agree	Agree	Disagree	Strongly disagree	I don’t know
1	Using dynamic software is a new experience for me.					
2	I enjoyed learning with dynamic geometry activities.					
3	I have learned a lot using dynamic geometry software.					
4	It is useful to learn proof using dynamic geometry activities.					
5	Dynamic geometry activities facilitated geometry proof learning.					
6	My interest to proof learning increased using dynamic geometry activities.					
7	Dynamic geometry software enables me to get the idea of proof easier.					

N	Statement	Strongly agree	Agree	Disagree	Strongly disagree	I don't know
8	Using dynamic geometry activities facilitate completing proof step by step.					
9	It is useful to find help when needed to complete proving.					
10	Dynamic geometry activities facilitate conjecturing throughout experimenting.					
11	It is easier to formulate and understand a geometry statement using dynamic geometry activities.					
12	I found the daily-life dynamic geometry activities are useful.					
13	Dynamic geometry activities enable me to collaborate with my colleagues, which facilitate geometry proof learning.					
14	Dynamic geometry activities are not only one time presentations but also give different possibilities to understand.					
15	Dynamic geometry activities allow me to learn in my own learning tempo.					
16	I have learned better than the teacher can do without this new approach.					
17	I found it is useful to get support from the teacher during learning with dynamic geometry activities.					
18	Dynamic geometry activities make geometry class boring.					
19	Dynamic geometry activities make geometry class enjoyable.					

N	Statement	Strongly agree	Agree	Disagree	Strongly disagree	I don't know
20	Teacher allows me to recognize the properties of geometry construction using dynamic geometry software better than stable drawing on the blackboard.					

Further Comments

Thank you for your participating.

Appendix F: Entrance Test

Entrance Test

Dear Student!

This entrance test is a part of educational research. It aims at examining whether you have the prerequisite geometric knowledge that enables you to participate in the suggested geometric activities.

Any personal information obtained from this test will be confidential. Moreover, to feel more comfortable, write your name if you want or write a code that you may use again.

Please consider that:

1. Read every question carefully before responding.

2. Choose the correct answer from a), b), c), or d)

You will have **30 minutes** to complete the test.

Name or Code (Please remember it!). .... **Gender**: ....

1. The perpendicular bisector is

    a) a line cuts a line-segment into two equal parts at 90°.
    b) a line intersects another line to form a 90° angle.
    c) a line forms with a segment a right angle.
    d) the longest side in a right angled triangle.

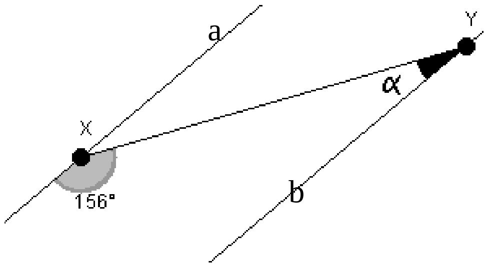
2. Two triangles are congruent if

    a) they have a congruent side.
    b) they have two equal angles in measure.
    c) their corresponding angles are equal in measure.
    d) their corresponding sides are equal length.

3. In the opposite figure, line a is parallel to line b.

   The measure of angle α is

   a) 34°
   b) 24°
   c) 114°
   d) 14°

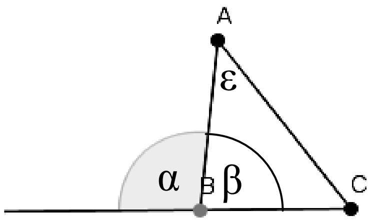


4. If two lines in a plane that ………. intersect, then they are **parallel**.

   a) sometimes
   b) do not
   c) always
   d) rarely

5. In the opposite figure, the measure of angle α is:

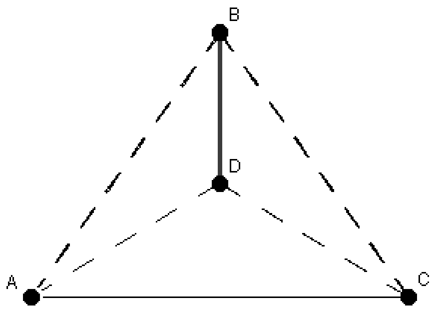
   a) 90°
   b) m(∠β)
   c) m(∠ε) + m(∠β)
   d) 180° – m(∠β)



6. In the opposite figure, AB = BC.

   and AD = CD. Which statement from the following is not true?

   a) Δ ADB ≡ Δ CDB (WWW)
   b) Δ ADB ≡ Δ CDB (SWS)
   c) Δ ADB ≡ Δ CDB (SSS)
   d) Δ ADB ≡ Δ CDB (WSW)

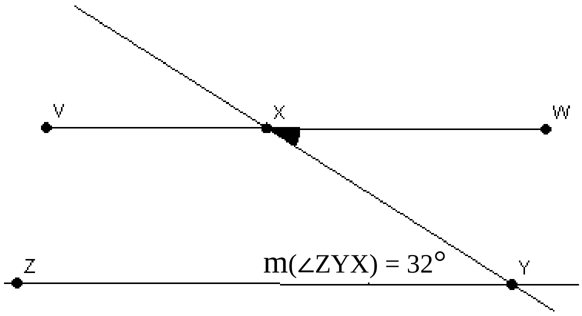


7. A possible definition of the bisector of an interior angle is ...

   a) a ray passes through the angle's vertex.
   b) a ray divides the angle into two angles.
   c) a straight line passes through two vertically opposite angles dividing them into four equal parts.
   d) a ray started at the vertex of the angle and passes through another point which is equidistant from angle's sides.

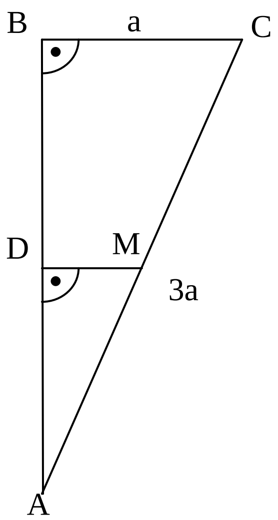
8. In the opposite figure, $\overline{VW}$ is parallel to $\overline{YZ}$. So that, m(∠WXY) equals:

   a) 148°
   b) 58°
   c) 23°
   d) 32°



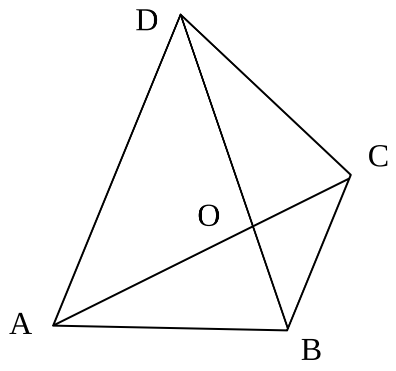
9. In the opposite figure, M is the midpoint of $\overline{AC}$. which statement is not true?

   a) AM = $^{3}/_{2}$ a
   b) AM = $^{3}/_{2}$ AD
   c) AB = 2a $\sqrt{2}$
   d) AD = $^{1}/_{2}$ AB



10. In the opposite figure, ABCD is an isosceles trapezoid, in which CD = AB

which statement is not true?



a) BC = ½ AD
b) AO = DO
c) AC = BD
d) BO = OC

Thank you

Appendix G: Geometric Achievement Test

Geometric Achievement Test

Dear Student!

This test is a part of educational research. It aims at examining the new geometric knowledge you have learned.

Any personal information obtained from this test will be confidential. Moreover, to feel more comfortable, write your name if you want or write a code that you may use again.

Please consider that:

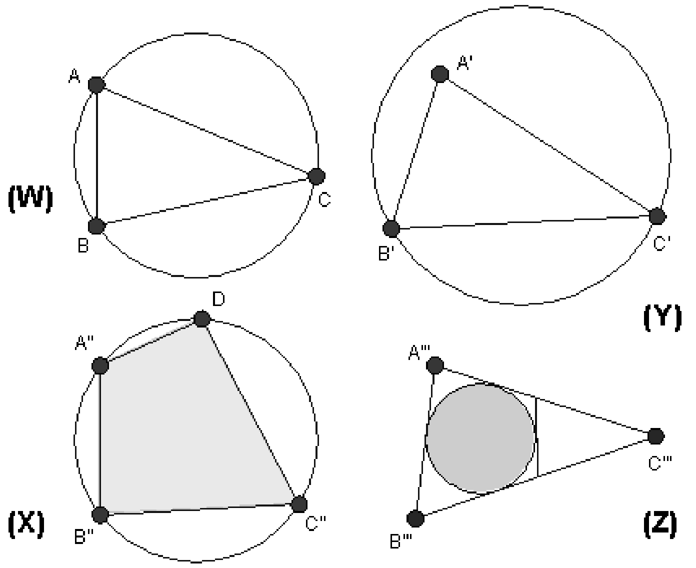
1. Read every question carefully before responding.
2. Choose the correct answer from a), b), c), or d)
3. If you asked to draw or prove, please use the available space provided after the question.

You have **45 minutes** to complete the test.

Your name or a code (Please remember it!). .... **Gender**: ....

1. In the opposite constructions, which one would show the circumcircle of a triangle?

   a) Construction (W)
   b) Construction (X)
   c) Construction (Y)
   d) Construction (Z)



2. In the above constructions, whi

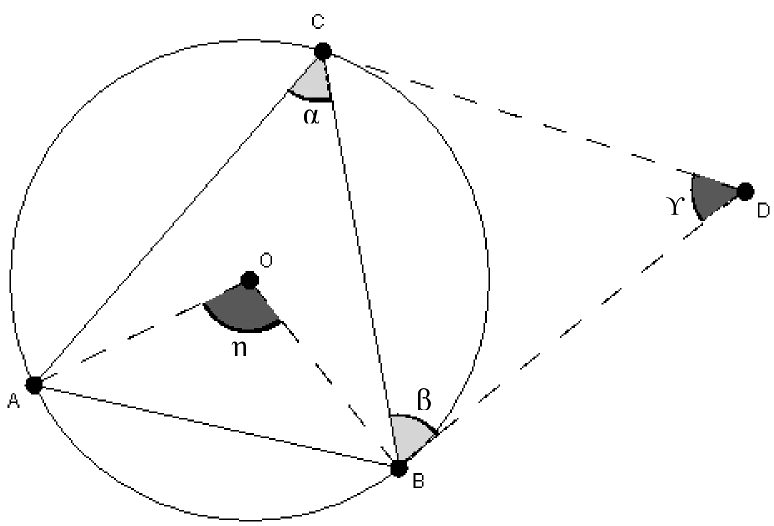
   a) Construction (W)
   b) Construction (X)
   c) Construction (Y)
   d) Construction (Z)

3. In the above constructions, which one would show an inscribed circle?

   a) Construction (W)
   b) Construction (X)
   c) Construction (Y)
   d) Construction (Z)

4. In the opposite figure, which of the following angles would show an inscribed angle?

   a) angle α
   b) angle β
   c) angle η
   d) angle ϒ

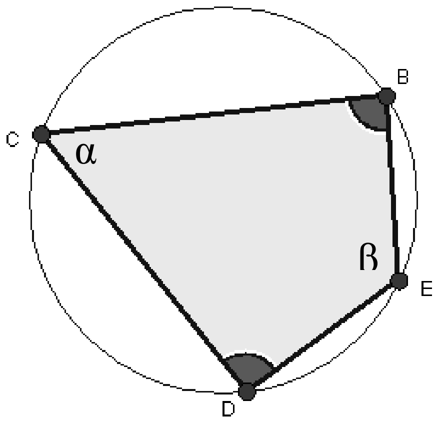


5. In the above figure, which of the following angle is a central angle?

   a) angle α
   b) angle β
   c) angle η
   d) angle ϒ

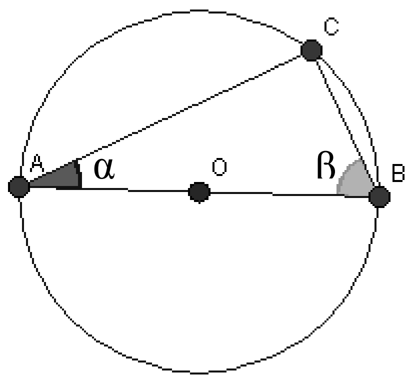
6. In the opposite figure, the sum of m(∠B) and m(∠D) equals:

   a) 270°
   b) 180°
   c) 3 |∢(α)|
   d) |∢(β)|



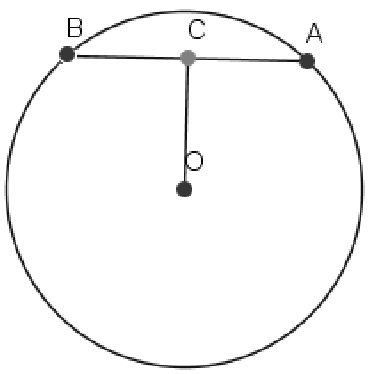
7. In the triangle ABC, the sum of m(∠α) + m(∠β) equals:

   a) 60°
   b) 120°
   c) m(∠BCA)
   d) m(∠BOA)

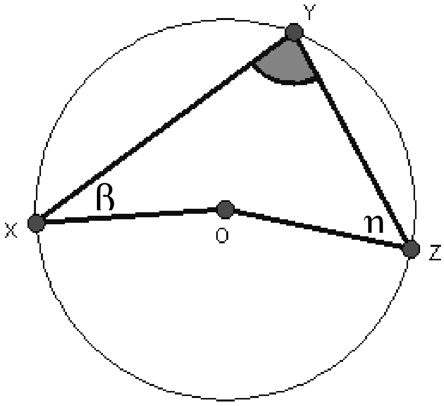


8. In the opposite figure, if $\overline{CO} \perp \overline{AB}$, then

   a) <
   b) >
   c) =
   d) ≠

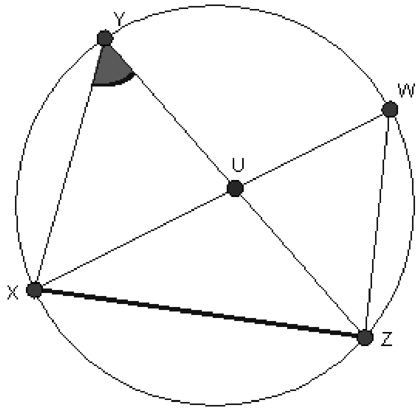


9. In the opposite figure, m(∠ZYX) equals:



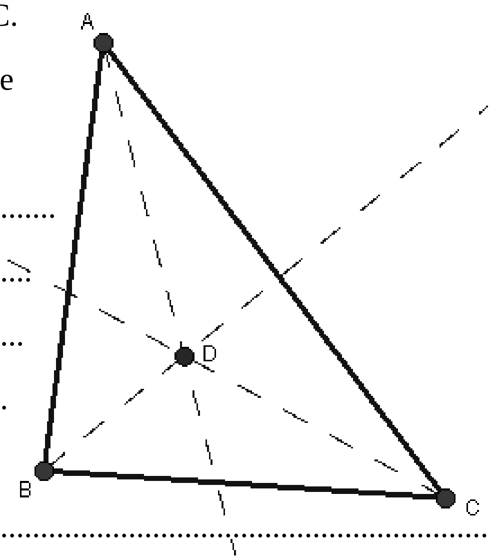
a) 90°
b) m(∠ZOX) / 2
c) half the measure of reflex angle ∠ZOX
d) 180° – [|∠(β) + |∠(η)|]

10. In the opposite figure, m(∠ZYX) equals:



a) m(∠YUW)
b) m(∠ZUX) / 2
c) m(∠W) + m(∠U)
d) m(∠ZWX)

11. In the opposite figure, D is the intersection point of the three angle bisectors of the triangle ABC. Prove that D is the center of the inscribed circle of the triangle ABC.



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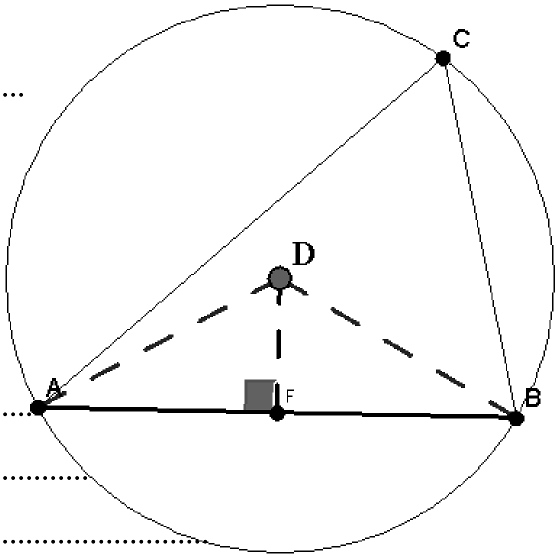
12. In the opposite figure, D is the circumcenter and $\overline{DF}$ is prependicular to the chord $\overline{AB}$.

Prove that F is the midpoint of $\overline{AB}$.



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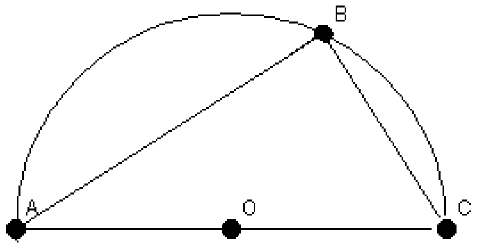
13. In the opposite figure, $\overline{AC}$ is the diameter of the circle O.

Prove that m(∠CBA) equals 90°.



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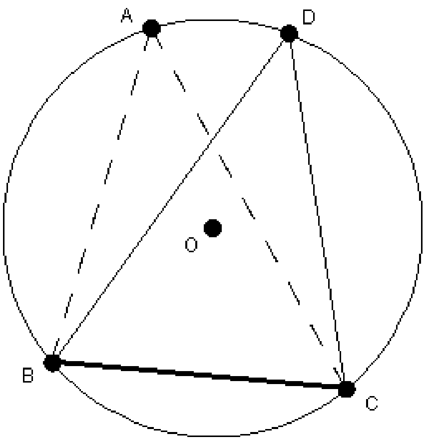
14. In the opposite figure, $\overline{CB}$ is a chord of the circle O.

Prove that m(∠CAB) = m(∠CDB).



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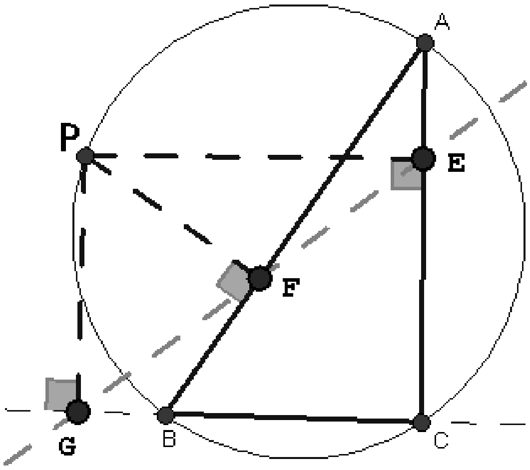
15. In the opposite figure, the point P lies on the circumcircle of the triangle ABC. $\overline{PE}$, $\overline{PF}$, and $\overline{PG}$ are perpendiculars to $\overline{AC}$, $\overline{AB}$, and $\overline{BC}$ respectively.

    Prove that the points E, F and G are collinear.



Thank you!

Appendix H: Worksheets

Worksheets

Name or ID code: (please remember your ID)............................... **Gender**: ....

- **Problem**

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- **Observation**

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- **Configurations**

- **Conclusions and Conjectures**

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- **Theorem**

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- **Configurations**

- **Proof Idea**

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- **Configurations**

- **Givens**

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- **Required to Prove**

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- **Configurations**

- **Proof**

Thank you!

Appendix I: Fragebogen zum Geometriebeweis

Fragebogen zum Geometriebeweis

Liebe Studentinnen und Studenten,

Ich möchte gerne wissen, ob du das Beweisen in der Geometrie für eine gute Aufgabe für das Lernen von Geometrie hältst oder nicht.

Alle Angaben in diesem Fragebogen werden vertraulich und geheim behandelt. Gebe deinen Namen oder einen Code an, den du auf den anderen Bögen wiederverwendest. Auf diesem Weg kann dich niemand identifizieren.

Bitte lese die unten stehenden Sätze genau durch, bevor du entscheidest! Beantworte jede Frage, es ist immer auch möglich "keine Angabe" anzukreuzen. Mache nur 1 Kreuz pro Zeile.

Teilnehmer ID : **(Bitte merken)** .... **Geschlecht**: ....

N	Sätze	Stimmt vollständig	Stimmt eher	Stimmt eher nicht	Stimmt gar nicht	Keine Angabe
1	Ich besuche gerne Geometrieveranstaltungen, wenn diese Beweisführungen beinhaltet.					
2	Geometrie ist mein mathematisches Lieblingsthema.					
3	Geometrische Beweise sind für mich einfacher als andere mathematische Themen.					
4	Ich bin frustriert, wenn ich eine Beweisführung nicht lösen kann und komme beim Weiterrechnen durcheinander.					
5	Ich finde Geometriebeweise so schwer, dass sie mir keine Freude machen.					
6	Ich brauche mehr Zeit, um geometrische Beweise zu verstehen, als für mathematische Inhalte ohne Beweise.					
7	Ich finde es oft schon schwer, geometrische Aussagen überhaupt zu verstehen.					

N	Sätze	Stimmt vollständig	Stimmt eher	Stimmt eher nicht	Stimmt gar nicht	Keine Angabe
8	Wenn mir jemand die Beweisidee gibt, kann ich damit normalerweise einen Beweis führen.					
9	Durch Diskussionen mit Dozent und Kommilitonen kann ich eine Beweisvermutung anstellen.					
10	Obwohl ich meine die Begründung für einen geometrischen Sachverhalt verstanden zu haben, fällt es mir schwer, diese zu erläutern.					
11	Wenn ich Schwierigkeiten habe, einen Beweis, den ich eigentlich verstanden habe, aufzuschreiben, fehlt mir dazu die passende Unterstützung.					
12	Geometrische Beweise geben mir persönlich tiefere Einsicht, weshalb eine geometrische Aussage allgemeingültig ist.					
13	Oft frage mich, warum ich etwas beweisen soll, wenn es schon längst bewiesen worden ist.					
14	Die Anwendung eines geometrischen Satzes ist meiner Meinung nach wichtiger, als ihn beweisen zu können.					
15	Ich habe gelernt, nach Verallgemeinerungen geometrischer Aussagen zu suchen.					
16	Es ist mir bei geometrischen Beweisaufgaben klar, was als Voraussetzung gegeben ist und was bewiesen werden soll.					
17	Ich kann einen geometrischen Beweis in Zusammenarbeit mit anderen Studenten zu Ende führen.					
18	Ich finde es frustrierend, wenn mein Ergebnis nicht mit dem des Lehrers oder anderen Studenten übereinstimmt.					
19	Der Beweis wird selten in Interaktion zwischen Lehrer und Studierenden erarbeitet.					

N	Sätze	Stimmt vollständig	Stimmt eher	Stimmt eher nicht	Stimmt gar nicht	Keine Angabe
20	Es stört mich, wenn auf meine Fragen oder meine Mitarbeit nicht eingegangen wird.					
Zusätzliche Anmerkungen:						

Danke für Ihre Mitarbeit.

Appendix J: Fragebogen zum Lernen mit dem Computer

Fragebogen zum Lernen mit dem Computer

Liebe Studentinnen und Studenten,

ich möchte gerne wissen, ob du Computer-Programme zum Lernen gut findest oder nicht.

Alle Angaben in diesem Fragebogen werden vertraulich und geheim behandelt. Gebe deinen Namen oder einen Code an, den du auf den anderen Bögen wiederverwendest. Auf diesem Weg kann dich niemand identifizieren.

Bitte lese die unten stehenden Sätze genau durch, bevor du entscheidest.

Beantworte jede Frage, es ist immer möglich "keine Angabe" anzukreuzen.

Bitte mache nur 1 Kreuz pro Zeile.

Teilnehmer ID : **(Bitte merken)** ....   **Geschlecht**: ....

N	Sätze	Stimmt vollständig	Stimmt eher	Stimmt eher nicht	Stimmt gar nicht	Keine Angabe
1	Das Lernen mit dem Computer macht mir Spaß.					
2	Ich brauche Hilfe bei der Benutzung des Computers.					
3	Für mich gibt es keine Möglichkeit, den Computer regelmäßig zum Lernen zu benutzen.					
4	Aus Angst, Fehler zu machen, benutze ich den Computer ungern.					
5	Der Computer macht das Lernen langweilig.					
6	Ich kann ohne den Computer besser lernen.					
7	Lernen am Computer ist zeitaufwendiger als herkömmliche Methoden.					
8	Die Computersoftware motiviert mich zum Lernen.					

N	Sätze	Stimmt vollständig	Stimmt eher	Stimmt eher nicht	Stimmt gar nicht	Keine Angabe
9	Der Computer hilft mir durch Veranschaulichung abstrakte Inhalte zu verstehen.					
10	Ich benutze den Computer auch zu Hause zum Lernen.					
11	Ich finde, dass das Arbeiten mit dem Computer die Motivation steigert.					
12	Der Lehrer benutzt den Computer, um den Unterricht zu verbessern.					
13	Inhalte des Unterrichts können durch den Computer verständlicher gemacht werden, als an der Tafel.					
14	Der Computer macht Aufgaben lebensnäher.					
15	Das Lernen mit dem Computer lässt Raum für Diskussionen im Klassenzimmer.					

Haben Sie schon mit Mathematikprogrammen gearbeitet? ☐ Ja ☐ Nein

Falls ja, mit welcher Software?

Welche Vorteile hat diese Software?

Danke für Ihre Mitarbeit.

Appendix K: Fragebogen zum Einsatz dynamischer Aktivitäten beim Beweisen u. im Geometrieunterricht

Fragebogen zum Einsatz dynamischer Aktivitäten beim Beweisen u. im Geometrieunterricht

Liebe Studentinnen und Studenten,

Ich möchte gerne wissen, ob dynamische Aktivitäten für dich hilfreich sind, um Beweise in der Geometrie zu verstehen.

Alle Angaben in diesem Fragebogen werden vertraulich und geheim behandelt. Gebe deinen Namen oder einen Code an, den du auf den anderen Bögen wiederverwendest. Auf diesem Weg kann niemand dich identifizieren.

Bitte lese die unten stehenden Sätze genau durch, bevor du entscheidest.

Beantworte jede Frage, es ist immer möglich "keine Angabe" anzukreuzen.

Bitte mache nur 1 Kreuz pro Zeile.

Teilnehmer ID : **(Bitte merken)** .... **Geschlecht**: ....

N	Sätze	Stimmt vollständig	Stimmt eher	Stimmt eher nicht	Stimmt gar nicht	Keine Angabe
1	Die Nutzung einer dynamischen Geometriesoftware ist neu für mich.					
2	Das Bearbeiten der dynamischen Aktivitäten hat mir Spaß gemacht.					
3	Ich habe mit Hilfe der dynamischen Aktivitäten viel gelernt.					
4	Dynamische Aktivitäten sind beim Lernen von Beweisführungen hilfreich.					
5	Dynamische Aktivitäten erleichtern das geometrische Beweislernen.					

N	Sätze	Stimmt vollständig	Stimmt eher	Stimmt eher nicht	Stimmt gar nicht	Keine Angabe
6	Die Arbeit mit dynamischen Aktivitäten hat mein Interesse am Beweislernen gesteigert.					
7	Die dynamischen Aktivitäten helfen mir, die Beweisidee einfacher zu erkennen und umzusetzen.					
8	Die dynamischen Aktivitäten vereinfacht die Beweisführung Schritt für Schritt.					
9	Ich finde es hilfreich, dass es bei dynamischen Aktivitäten die Möglichkeit gibt, weitere Hilfen zu erhalten, wenn ich nicht weiter komme.					
10	Das eigene Experimentieren mit dynamischen Aktivitäten erleichtert es mir, Vermutungen zu treffen.					
11	Durch die dynamischen Aktivitäten kann ich leichter geometrische Aussagen formulieren und verstehen.					
12	Ich finde dynamische Aktivitäten aus dem Alltag hilfreich.					
13	Durch die dynamischen Aktivitäten kann ich mit meinen Kommilitonen zusammen arbeiten, was das Beweislernen vereinfacht.					
14	Die dynamischen Aktivitäten begleiten mich bei jedem Schritt und zeigt mir mehrere Herangehensweisen.					
15	Mit Hilfe der dynamischen Aktivitäten kann ich Beweise in meinem eigenen Tempo lernen.					
16	Ich finde, dass man <u>mit</u> dynamischen Aktivitäten mehr lernt, als es ein Lehrer ohne könnte.					
17	Für mich war die Unterstützung durch den Lehrer beim Bearbeiten der dynamischen Aktivitäten hilfreich.					
18	Unterricht mit dynamischen Aktivitäten macht das Lernen langweilig.					

N	Sätze	Stimmt vollständig	Stimmt eher	Stimmt eher nicht	Stimmt gar nicht	Keine Angabe
19	Unterricht mit dynamischen Aktivitäten macht das Thema Geometrie angenehm, besser.					
20	Die dynamische Geometriesoftware ermöglicht es mir, die Eigenschaften geometrischer Konstruktionen besser zu erkennen, als an der Tafel.					

Zusätzliche Anmerkungen:

Danke für Ihre Mitarbeit

Appendix L: Eingangsprüfung

Eingangsprüfung

Liebe Studentin, lieber Student!

Dies ist eine freiwillige Geometrieprüfung. Du solltest dir einen Überblick über deinen Wissensstand verschaffen.

Alle Angaben und deine Ergebnisse in diesem Test werden vertraulich und geheim behandelt. Gebe deinen Namen oder einen Code an, den du auf den anderen Bögen wiederverwendest. Auf diesem Weg kann niemand, dich identifizieren.

Bitte beachte:

3. Lese jede Frage sorgfältig durch, bevor du antwortest.
4. Bei allen Fragen muss aus mehreren Antworten eine ausgewählt werden.

   Bitte kreuze immer nur **eine** Antwort an.

Du hast **30 Minuten** zur Verfügung, um die Fragen zu beantworten.

Name oder Code (Diesen Code bitte merken!). .... **Geschlecht**: ....

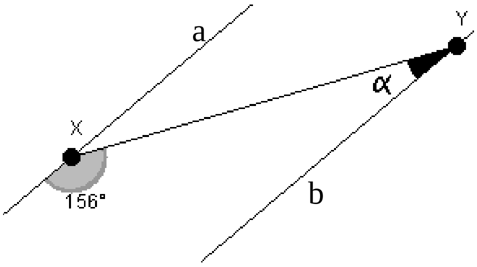
11. Die Definition der Mittelsenkrechten ist ...

    e) eine Gerade, die eine Strecke senkrecht halbiert.
    f) eine Gerade, die eine andere Linie kreuzt, sodass ein 90°-Winkel entsteht.
    g) eine Gerade, die mit einer Strecke einen rechten Winkel bildet.
    h) die längste Seite eines rechtwinkligen Dreiecks.

12. Zwei Dreiecke sind kongruent, wenn .....

    e) sie in einer Seite übereinstimmen.
    f) sie in zwei Winkeln übereinstimmen.
    g) sie in ihren drei Winkeln übereinstimmen.
    h) sie in ihren drei Seitenlängen übereinstimmen.

13. In der nebenstehenden Figur ist die Gerade a parallel zur Gerade b. Der Winkel α misst

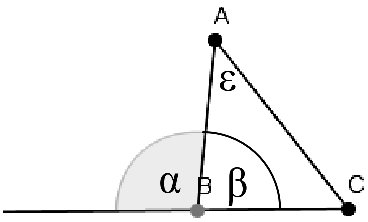


e) 34°
f) 24°
g) 114°
h) 14°

14. Zwei Geraden sind **parallel**, wenn sie in einer Ebene liegen und einander .... schneiden.

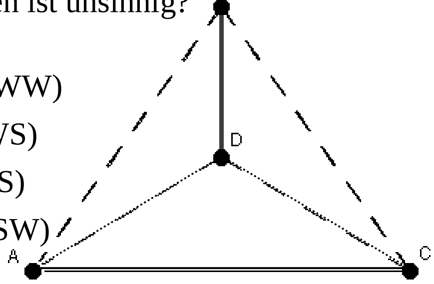
e) manchmal
f) nicht
g) immer
h) selten

15. In der nebenstehenden Figur mit drei sich kreuzenden Geraden gilt für den Winkel α sicher:



e) Er misst 90°
f) Er misst genauso viel wie β
g) Er misst ε + β
h) Er misst 180° - β

16. In der nebenstehenden Figur $|\overline{AB}| = |\overline{BC}|$. und $|\overline{AD}| = |\overline{CD}|$. Welche der folgenden Aussagen ist unsinnig?



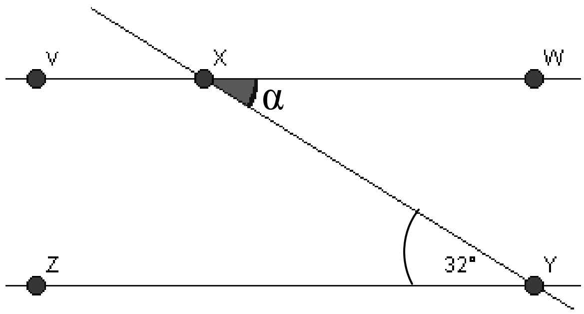
e) Δ ADB und Δ CDB kongruent sind nach (WWW)
f) Δ ADB und Δ CDB kongruent sind nach (SWS)
g) Δ ADB und Δ CDB kongruent sind nach (SSS)
h) Δ ADB und Δ CDB kongruent sind nach (WSW)

17. Eine mögliche Definition für Winkelhalbierende ist ...

    e) eine Halbgerade, die durch den Scheitelpunkt des Winkels läuft.
    f) eine Halbgerade, die den Winkel in zwei Teile teilt.
    g) der geometrische Ort aller Punkte, die von den beiden Schenkeln des Winkels den gleichen Abstand haben.
    h) eine Gerade, die durch zwei Scheitelwinkel läuft und den Winkel in vier gleiche Teile teilt.

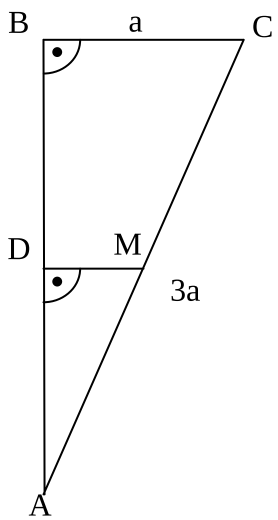
18. In der nebenstehenden Figur ist die Gerade $\overline{VW}$ parallel zur Gerade $\overline{YZ}$. Der Winkel α ist gleich:

    e) 148°
    f) 58°
    g) 23°
    h) 32°



19. In der nebenstehenden Figur sei $\overline{BC} = a$, $\overline{AC} = 3a$ und M der Mittelpunkt der Strecke $\overline{AC}$. Welche Gleichung stimmt nicht?

    e) $|\overline{AD}| = \frac{1}{2}\,|\overline{AB}|$
    f) $|\overline{AM}| = {}^{3}/_{2}\ a$
    g) $|\overline{AB}| = 2a\,\sqrt{2}$
    h) $|\overline{AM}| = {}^{3}/_{2}\ |\overline{AD}|$

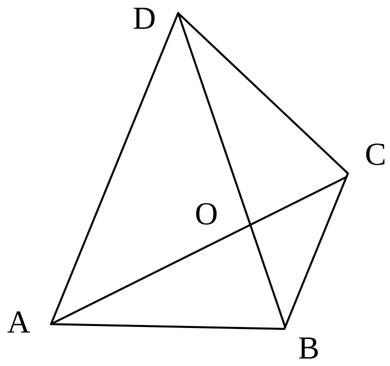


20. Das untenstehende *ebene* Viereck ABCD ist ein gleichschenkliges Trapez mit

$|\overline{DC}| = |\overline{AB}|$

Welche der folgenden Aussagen ist im Allgemeinen falsch?

e) $|\overline{BC}| = \frac{1}{2}\,|\overline{AD}|$
f) $|\overline{AO}| = |\overline{DO}|$
g) $|\overline{AC}| = |\overline{BD}|$
h) $|\overline{BO}| = |\overline{OC}|$



Vielen Dank

Appendix M: Geometrieprüfung

Geometrieprüfung

Liebe Studentin, lieber Student!

Dies ist eine freiwillige Geometrieprüfung. Du solltest dir einen Überblick über deinen Wissensstand verschaffen.

Alle Angaben und deine Ergebnisse in diesem Test werden vertraulich und geheim behandelt. Gebe deinen Namen oder einen Code an, den du auf den anderen Bögen wiederverwendest. Auf diesem Weg kann niemand, dich identifizieren.

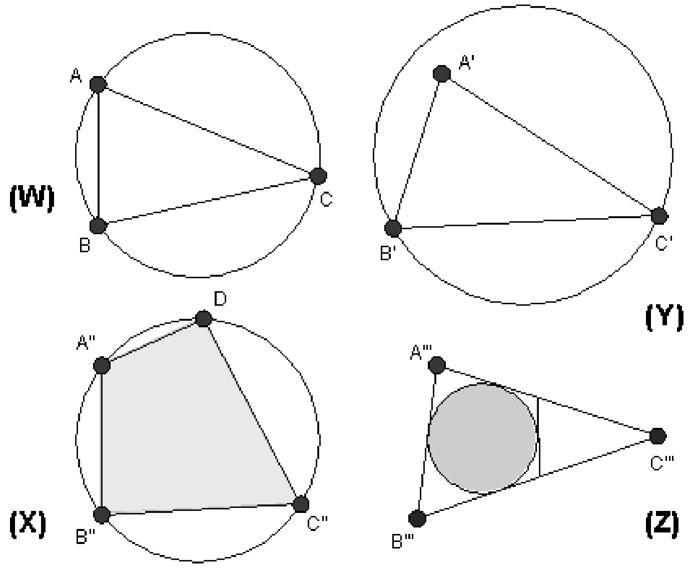
Bitte beachte:
1. Lese jede Frage sorgfältig durch, bevor du antwortest.
2. Bei einigen Fragen bist du auffordert, aus mehreren Antworten um eine auszuwählen. Bitte kreuze in diesen Fällen nur **eine** Antwort an.
3. Wo du aufgefordert wirst, eine Zeichnung anzufertigen oder eine Beweisführung zu machen, nutze bitte den dafür vorgesehenen freien Platz auf dem Prüfungsbogen unter der Frage.

Du hast **45 Minuten** zur Verfügung, um die Fragen zu beantworten.

Name oder Code (Diesen Code bitte merken!). .... **Geschlecht**: ....

1. Welche der nebenstehenden Konstruktionen zeigt den Umkreis eines Dreiecks?

    a) Konstruktion (W)
    b) Konstruktion (X)
    c) Konstruktion (Y)
    d) Konstruktion (Z)



2. Welche der oben abgebildeten Konstruktionen zeigt ein Sehnenviereck?

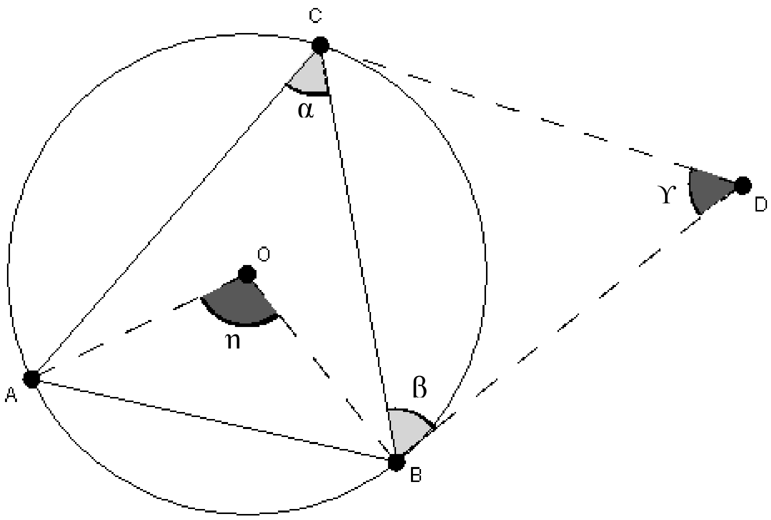
    a) Konstruktion (W)
    b) Konstruktion (X)
    c) Konstruktion (Y)
    d) Konstruktion (Z)

3. Welche der oben abgebildeten Konstruktionen zeigt den Innenkreis eines Dreiecks?

    a) Konstruktion (W)
    b) Konstruktion (X)
    c) Konstruktion (Y)
    d) Konstruktion (Z)

4. In der nebenstehenden Konstruktion: Welche der folgenden Winkeln ist ein Umfangswinkel?

   a) der Winkel α
   b) der Winkel β
   c) der Winkel η
   d) der Winkel ϒ

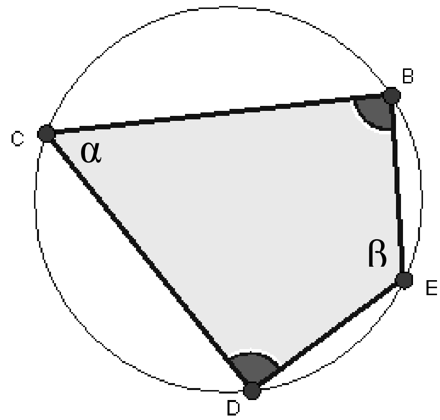


16. In der vorherigen Konstruction: Welche der folgenden Winkeln ist ein Mittelswinkel?

   a) der Winkel α
   b) der Winkel β
   c) der Winkel η
   d) der Winkel ϒ

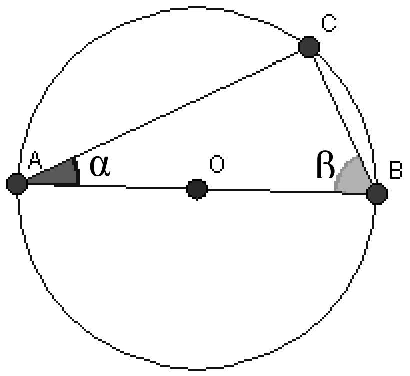
17. In der nebenstehenden Konstruction gilt für die Summe |∢(B)| + |∢(D)| :

   a) 270°
   b) 180°
   c) 3 |∢(α)|
   d) |∢(β)|



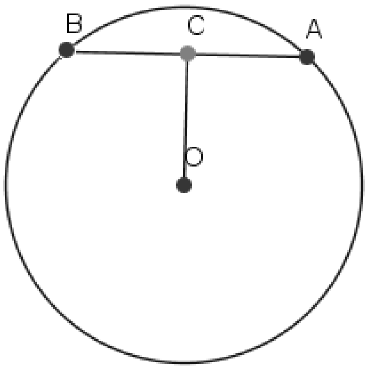
18. Im Dreieck ABC, ist die Summe von |∢(α)| + |∢(β)| ist:

a) 60°
b) 120°
c) |∢(BCA)|
d) |∢(BOA)|



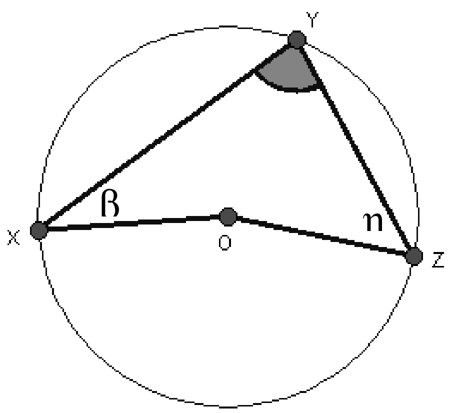
19. Wenn in der nebenstehenden Figur $\overline{CO} \perp \overline{AB}$ ist, dann ist $|\overline{AC}|$ ……. $|\overline{BC}|$.

a) <
b) >
c) =
d) ≠



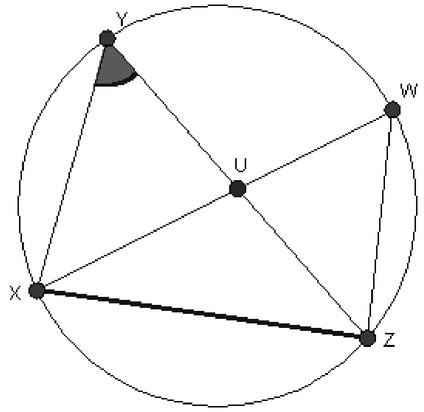
20. In der nebenstehenden Figur, misst der Winkel α :

a) 90°
b) |∢(ZOX)| / 2
c) Überstumpfer |∢(ZOX)| / 2
d) 180° – [|∢(β)| + |∢(η)|]

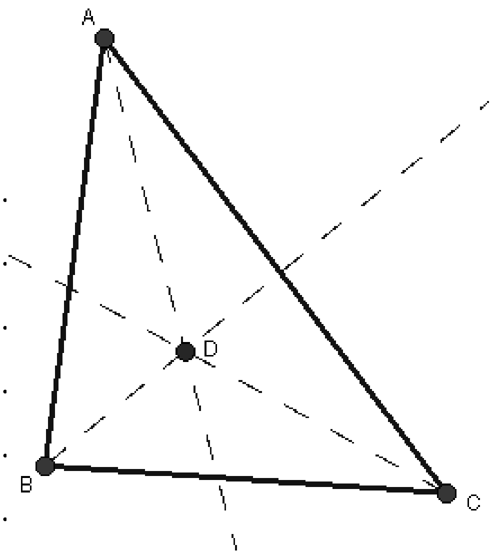


21. In der nebenstehenden Figur, ist der Winkel ∢(ZYX) gleich groß wie:

a) |∢(YUW)|
b) |∢(ZUX)| / 2
c) |∢(W)| + |∢(U)|
d) |∢(ZWX)|



22. In der nebenstehenden Figur, ist D Schnittpunkt der drei Winkelhalbierenden eines Dreiecks.
Zeige, dass dieser Schnittpunkt ist
der Mittelpunkt des Inkreises des Dreiecks ist.



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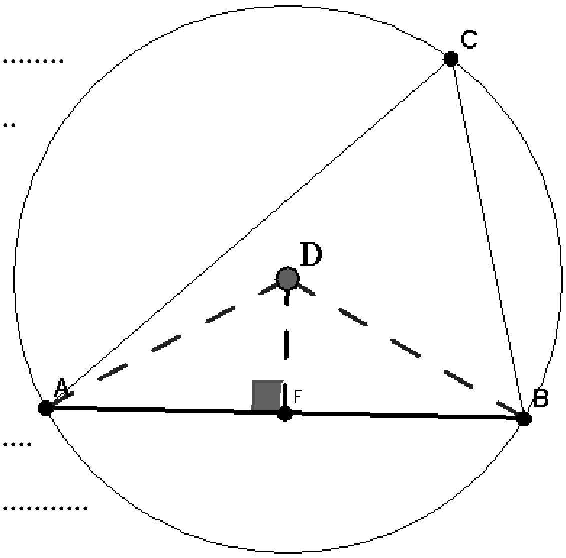
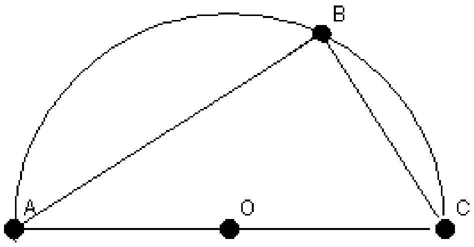
23. In der nebenstehenden Figur fällt vom Mittelpunkt des Kreises D eine Senkrechte auf die Sehne $\overline{AB}$. Zeige, dass F der Mittelpunkt der Sehne ist.

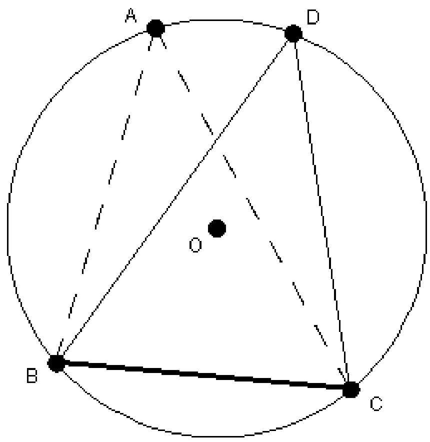


24. In der nebenstehenden Figur ist $\overline{AC}$ der Durchmesser des Kreises O.

Zeige, dass der Winkel ∢(CBA) ein rechter Winkel ist.

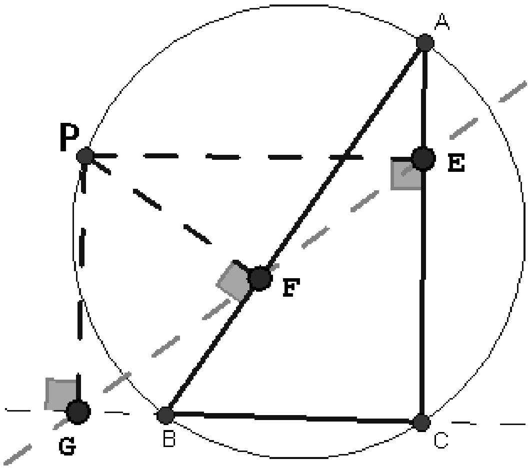


25. In der nebenstehenden Figur ist $\overline{CB}$ eine gemeinsame Sehne. Zeige, dass die Winkel ∢(CAB) und ∢(CDB) gleich groß sind.



26. In der nebenstehenden Figur liegt der Punkt P "ein beliebiger Punkt" auf dem Umkreis des Dreiecks ABC. Die Punkte E, F und G sind die Fußpunkte des Lotes von P auf den Seiten oder Seitenverlängerungen des Dreiecks.

Zeige, dass die Punkte E, F und G auf einer Gerade liegen.



Vielen Dank!

Appendix N: Arbeitsblätter

Arbeitsblätter

Name oder Code (Diesen Code bitte merken!). .... **Geschlecht**: ....

- **Problem**

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- **Beobachtung**

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- **Skizzen**

- **Ergebnisse und Vermutungen**

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- **Lehrsatz**

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- **Skizzen**

- **Beweis-Idee**

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- **Skizzen**

- **Gegeben**

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- **Ziel**

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....

- **Skizzen**

- **Beweis**

Vielen Dank!